Solar Physics & Space Plasma Research Center (SP²RC)

Fundamentals of MHD in Space Research II

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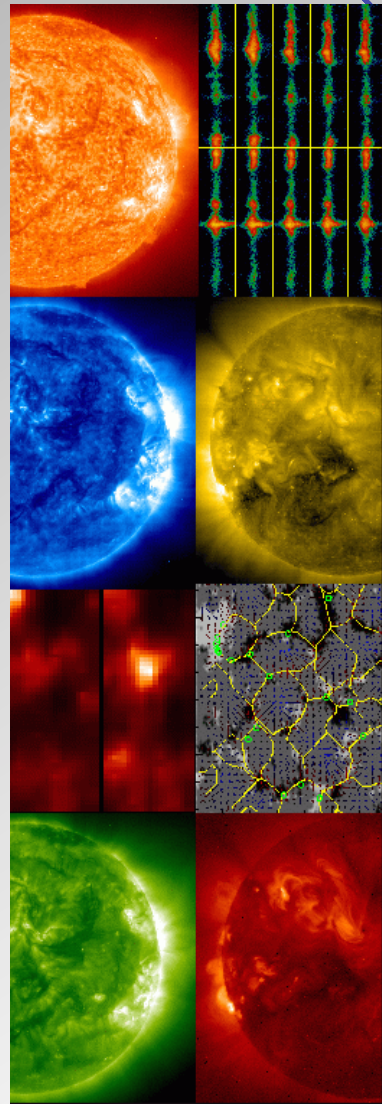


The Outline

- Introduction
- Observations of MHD waves
- Linear and (some) non-linear MHD waves
- Resonant flow instabilities
- Selected topics (stratification, thin flux tubes, [auto]solitons, applications)
- Conclusions

Why bother: "Big questions"

- What is the basis of stability and dynamics of solar atmospheric and ST structures?

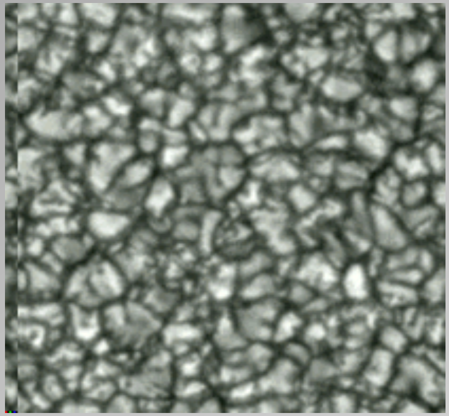
- What mechanisms are responsible for heating in the solar atmosphere up to several million K?

- What accelerates the solar wind up to measured speeds exceeding 700 km/s?

- What are the physical processes behind the enormous energy releases (e.g. flares, substorms)?

Why bother: "Big questions"

Coronal heating: Energy source for coronal heating: kinetic energy of convection zone



movie: Goran Scharmer/SVST

Waves ?

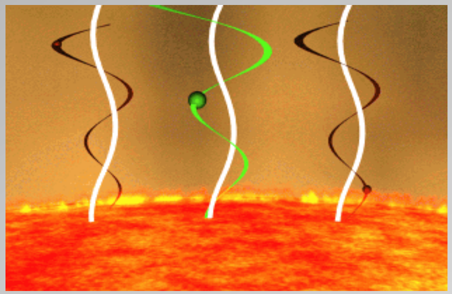


image: Dana Berry/NASA

Current sheets ?

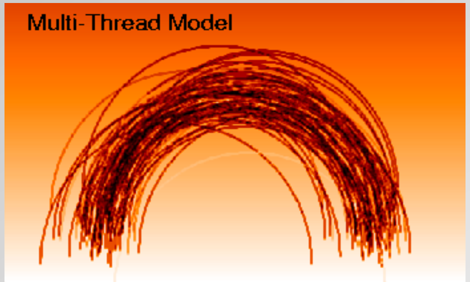


image: Marcus Aschwanden

Magnetic reconnection ?

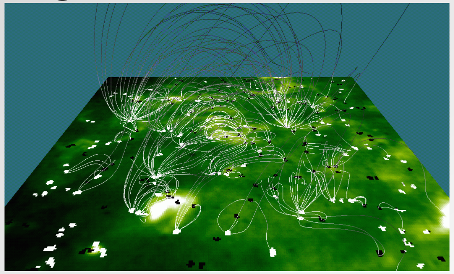


image: Neal Hurlburt/Karel Schrijver

What are MHD waves?

- How do we communicate in MHD? **MHD is kind!**

- **MHD waves** are **propagating perturbations** of **magnetic field**, **plasma velocity** and **plasma mass density**, described by the MHD (single fluid approximation) set of equations, which connects the magnetic field B, plasma velocity v , kinetic pressure p and density ρ.

- **Non-relativistic** approximation

Why to study MHD waves?

MHD waves are believed to play a crucial role in the dynamics and structure of the solar interior, in the entire solar atmosphere (sunspots, chromosphere, TR, corona, solar wind) and in Earth' magnetosphere. MHD waves are associated with

- **the evolution and development of plasma perturbations,**
- **the transfer of plasma energy and momentum,**
- **plasma heating / acceleration,**
- **diagnostics of magnetised plasma**
- **helioseismology, solar atmospheric (magneto) seismology, magnetosphere seismology.**
- **Also, we use it because simply they are there and affect us!**

Magnetic coupling: dynamic STS

- Photosphere – chromosphere – TR – corona (inluding solar wind) – magnetosphere – Earth's upper atmosphere are all **magnetically coupled**.
- Very highly **structured** and **dynamic**.

MHD seismology is a perfect tool to study this coupled, dynamic an structured system.

Two (biassed) particularly exciting aspects:

- **Influence of atmosphere** on global oscillations.
- **Role of *p* modes in the dynamics of the atmosphere!** (Not yet explored.)

Structured solar atmosphere

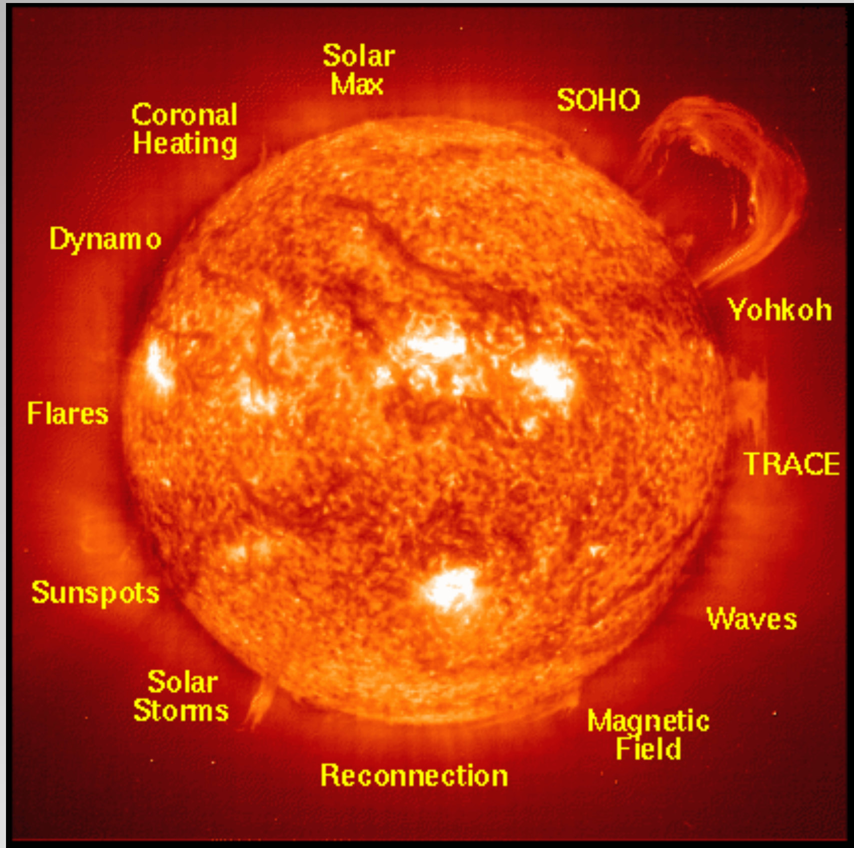
The corona is highly structured in magnetic field, in plasma density and in temperature. Steady corona.

There are two main classes of coronal structures:

• **Closed structures**: loops (R~100-200 Mm) which are hot (~ 2-3x10^6 K) and dense (up to 7x10^{15} m^{-3}). Life time: hours-days. However, loop ensembles called active regions (ARs) can live much longer.

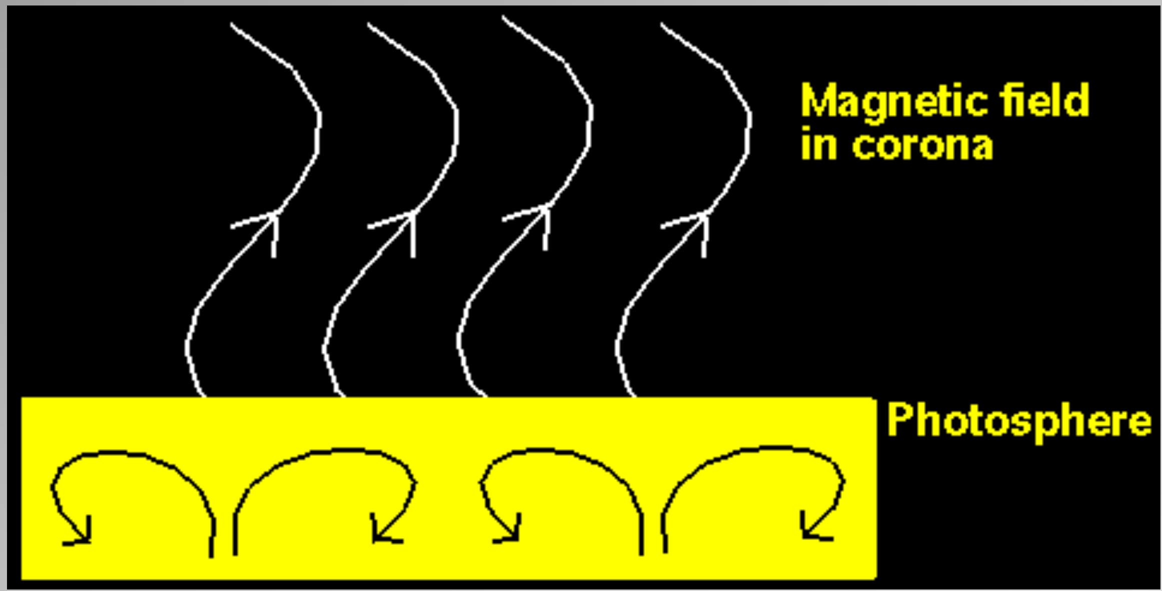
• **Open structures**: coronal holes, streamers, plumes inside the holes. Life time: days-weeks.

In addition, there are very dynamic plasma jets of various scales and speeds (erupting prominences, EEs, TRBs, etc.).



Courtesy: Unknown Nice Person

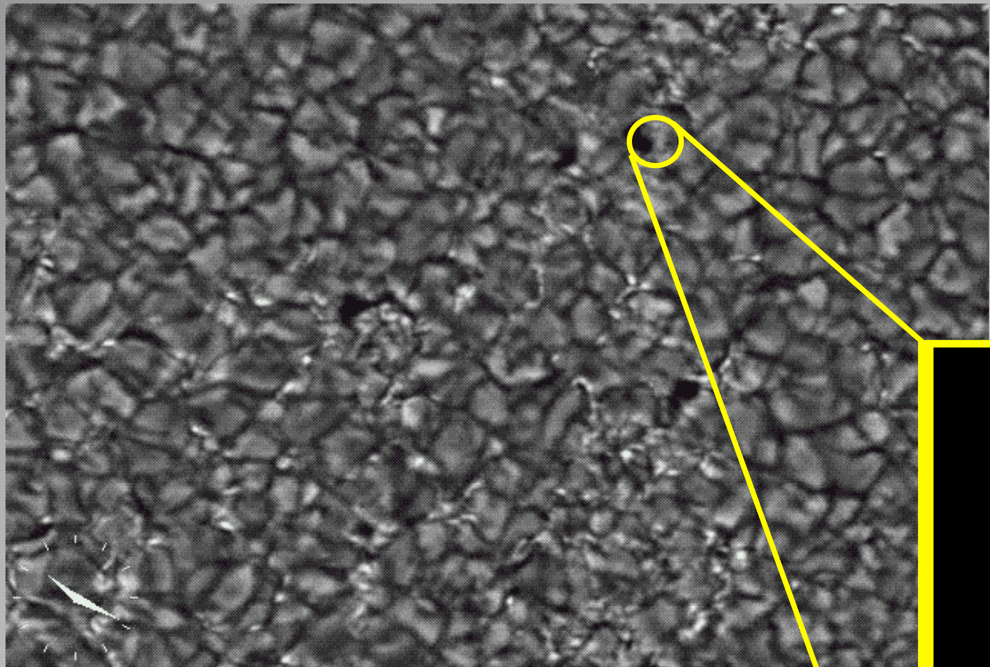
Do we "expect" solar MHD waves?



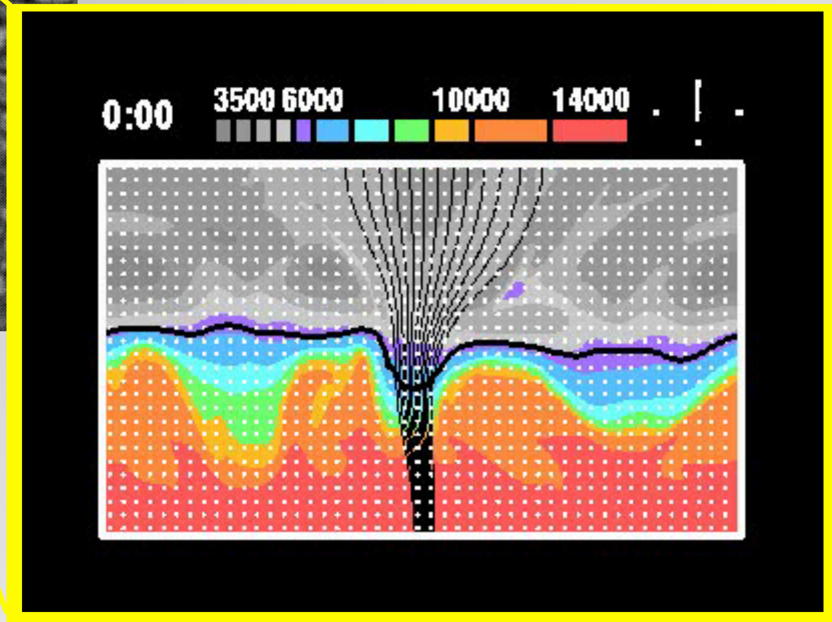
- Corona has *frozen in field condition*
- Magnetic field rooted into turbulent photosphere
- Generates "waves" that dump energy in corona
- Alfvén/(Slow/Fast) Magneto-acoustic waves

Do we "expect" solar MHD waves?

Dutch Open Telescope, La Palma
12. Sept. 1999 [Sütterlin & Rutten]



2D-simulation of a **flux tube** embedded in photospheric granulation (radiation-MHD)
[Steiner et al. (1997) ApJ 495, 468]

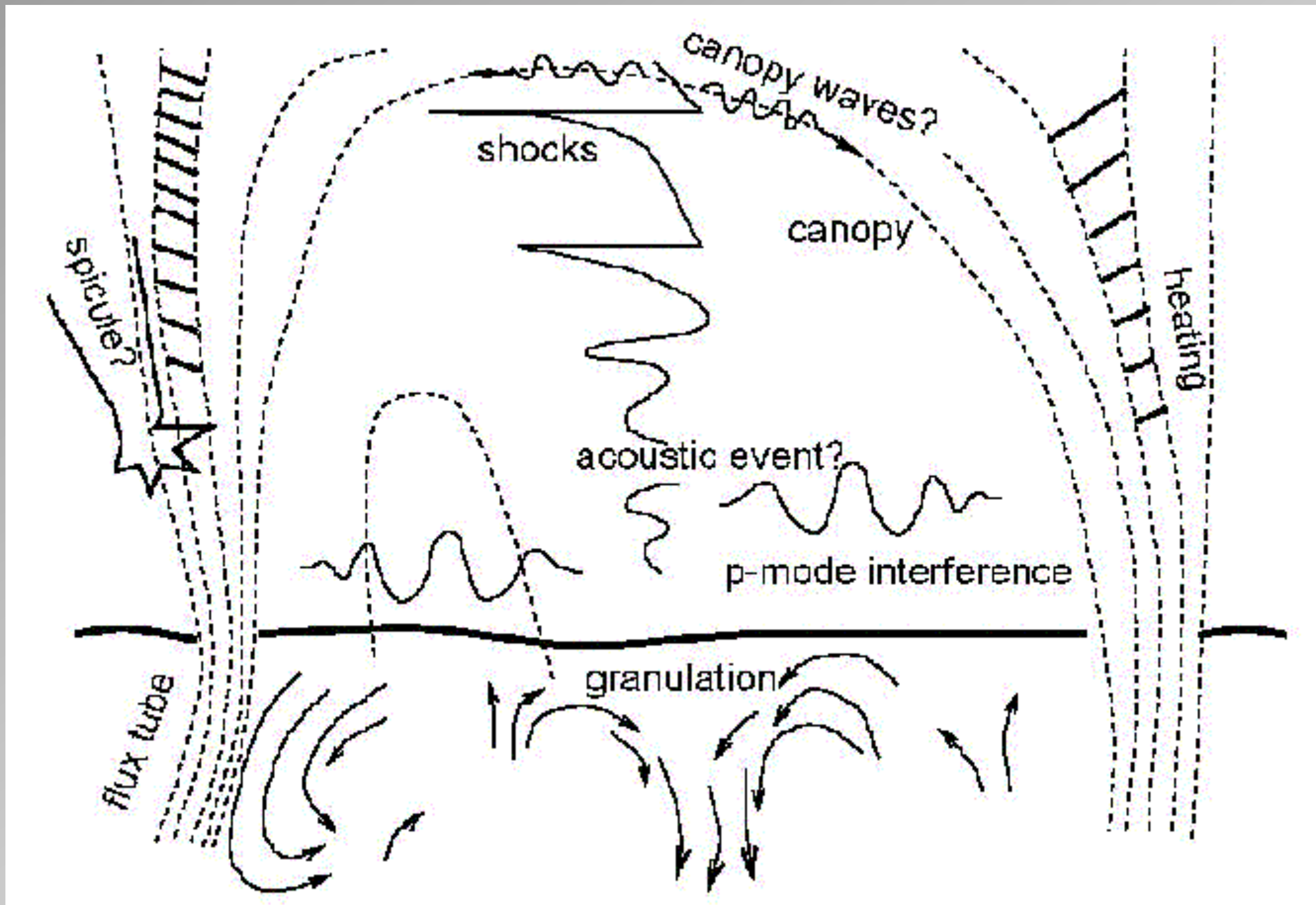


≈ 25 000 km x 38 000 km

observation in G-Band ≈ 430 nm
granulation (Ø ≈1000 km)
G-band bright points:
small magnetic flux tubes, which are brighter than their surrounding

≈ 2400 km x 1400 km, ≈ 18 min

Do we "expect" solar MHD waves?



Rutten, R., ASP-CS, 184, 181, 1999

Low atmosphere: role of underlying driver

Do we see MHD waves?

"Before SOHO and TRACE"

MHD waves and oscillations have been observed for a long time in radio and optic bands:

Prominence oscillations Periodic velocity and intensity oscillations with various periods: e.g. 1 hour, 3-5 min, 30 s. (They are seen from the Earth).

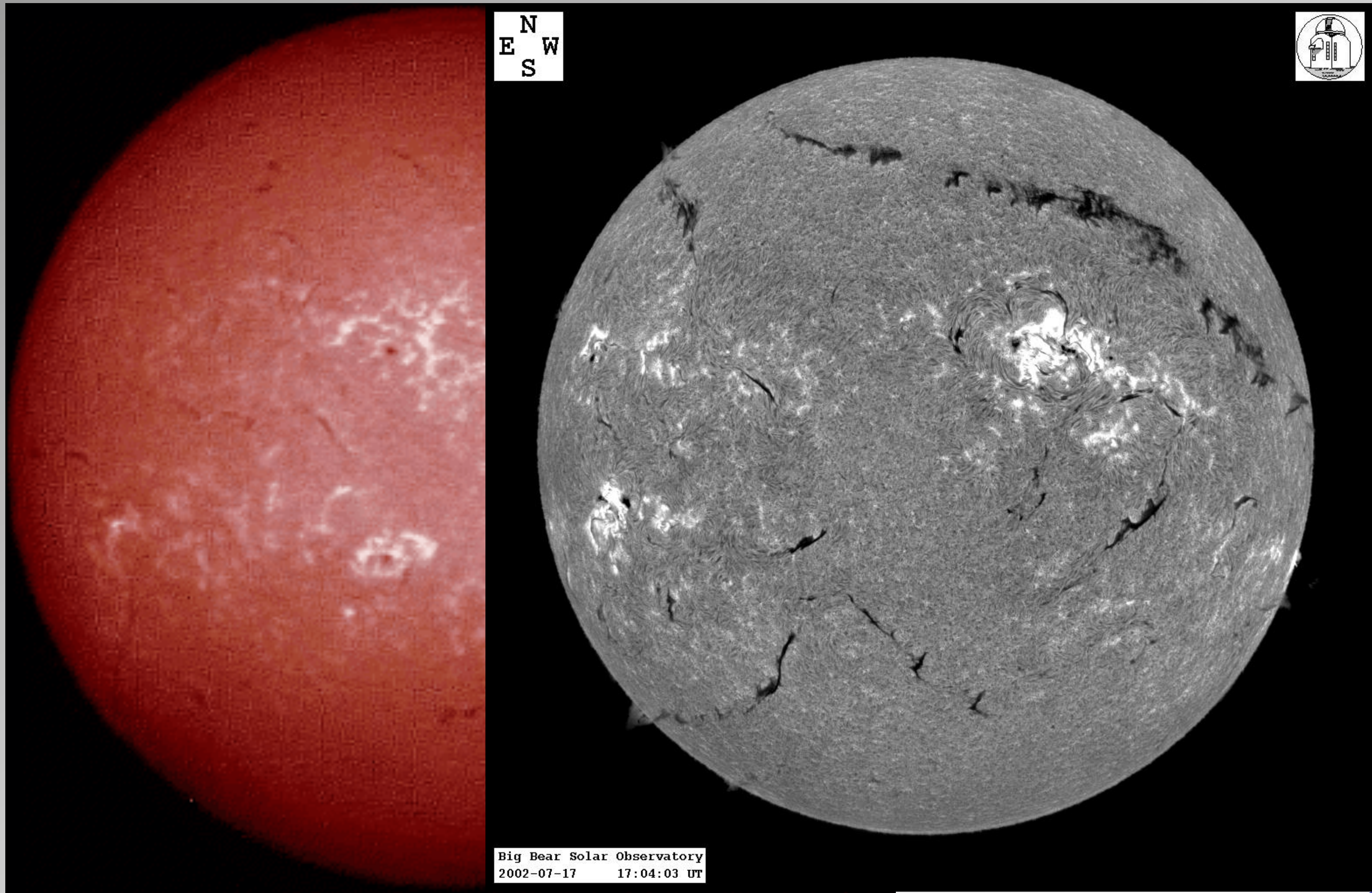
Radio pulsations Several periodicities were detected in the MHD band by the analysis of the coronal radio-emission. (See, e.g. Aschwanden 1987 for a review.)

Roberts et al. (1983): Type IV radio events have been observed and interpreted as fast waves trapped in loops. The idea of ***coronal seismology*** has been suggested for the first time.

EUV oscillations Probably, the first observations of MHD waves in the corona were reported by Chapman et al. (1972) with GSFC extreme-ultraviolet spectroheliograph on OSO-7 (spatial resolution was few arcsec, cadence time was 5.14 s). Mg VII, Mg IX and He II emission intensity periodicities at about 262 s have been detected.

Chromosphere: filaments

- Hydrogen alpha filter image
- Thickness ≈ 2500 km



Do we see MHD waves?

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Do we see MHD waves?

"Before SOHO and TRACE" (ctd)

Antonucci et al. (1984) using Harvard College Observatory EUV spectroheliometer on Skylab have detected oscillations in the C II, O IV, and Mg X emission intensity with periods of 117 s and 141 s.

Soft X-ray oscillations

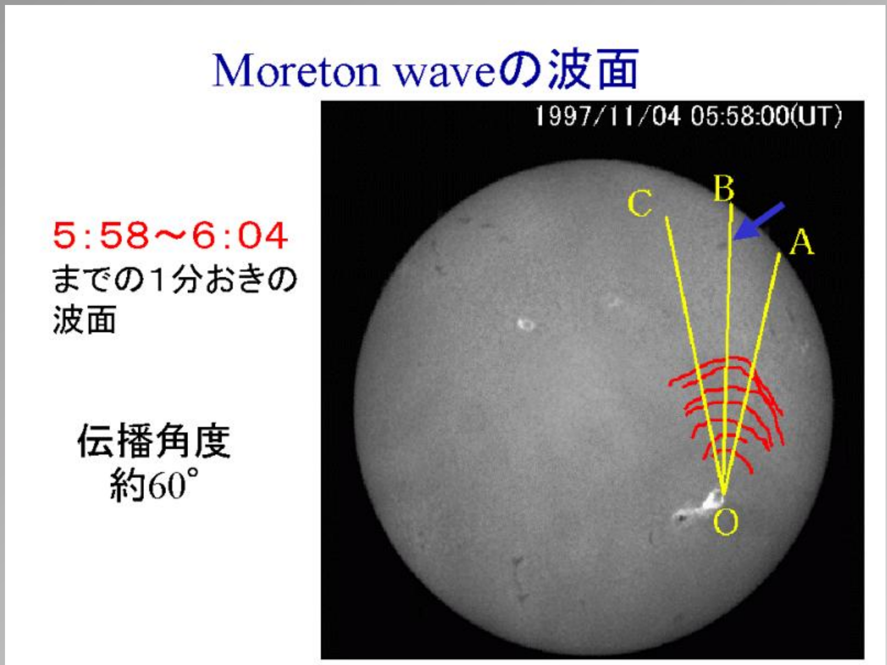
Harrison (1987) with Hard X-ray Imaging Spectrometer on SMM have detected soft X-ray (3.5-5.5 keV) pulsations of period 24 min (for six hours).

Moreton-waves

Large scale wave motions have been discovered in the corona in 1960!

Do we see MHD waves?

Moreton waves

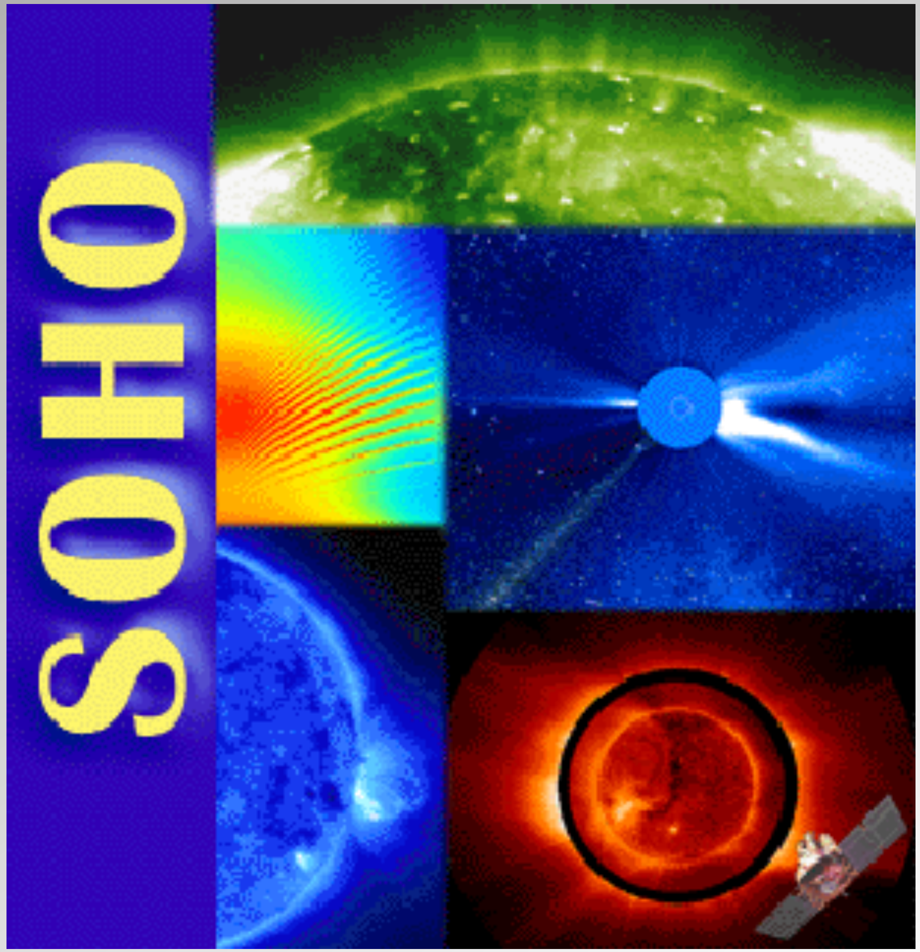


- Seen in Hα in the chromosphere at 10000 K (Moreton '60)
- Propagation speeds 450-2000 km/s, away from a flare site
- Propagate almost isotropically; confined to an arc rarely exceeding 120º
- Have been identified as the intersection of coronal shock waves (due to a flare) with the chromosphere (Uchida '68; '74)
- Are not seen to decelerate
- The generation mechanism has not been made clear yet

SOHO

The Solar and Heliospheric Observatory

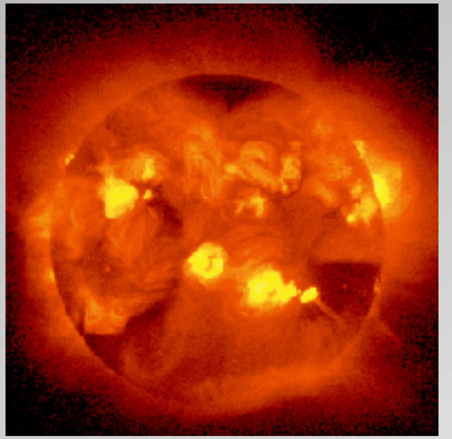
- Joint ESA and NASA project
- Suit of 12 instruments
- Launched in 1995
- 1.5 million km towards the Sun



Yohkoh & TRACE

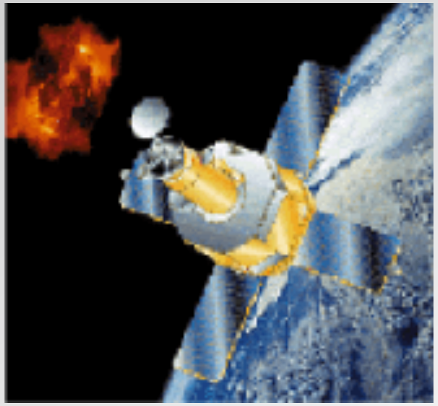
Yohkoh ("Sunbeam")

- Japan/UK/USA Mission
- Observed Sun in X-ray
- Launched in 1992

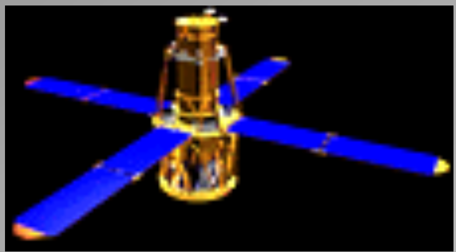


Transition Region and Coronal Explorer

- NASA Small Explorer
- EUV Mission
- Incredible resolution

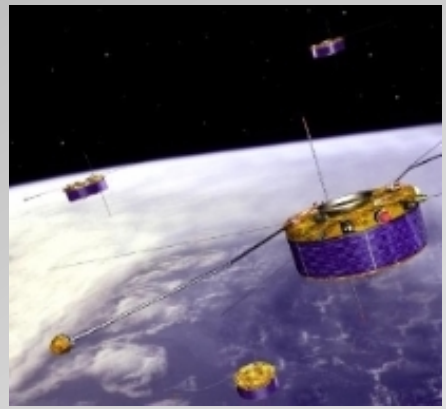


CLUSTER, RHESSI & Hinode



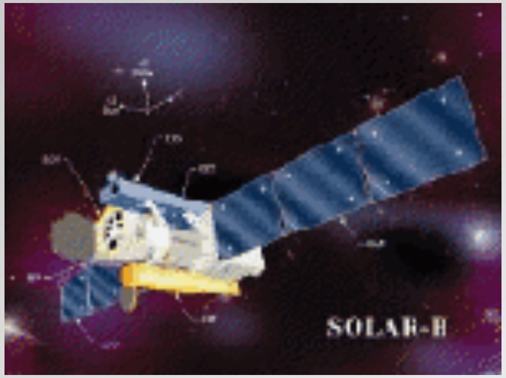
CLUSTER II

- Four satellite
- 3D magnetosphere
- July & August 2000



RHESSI

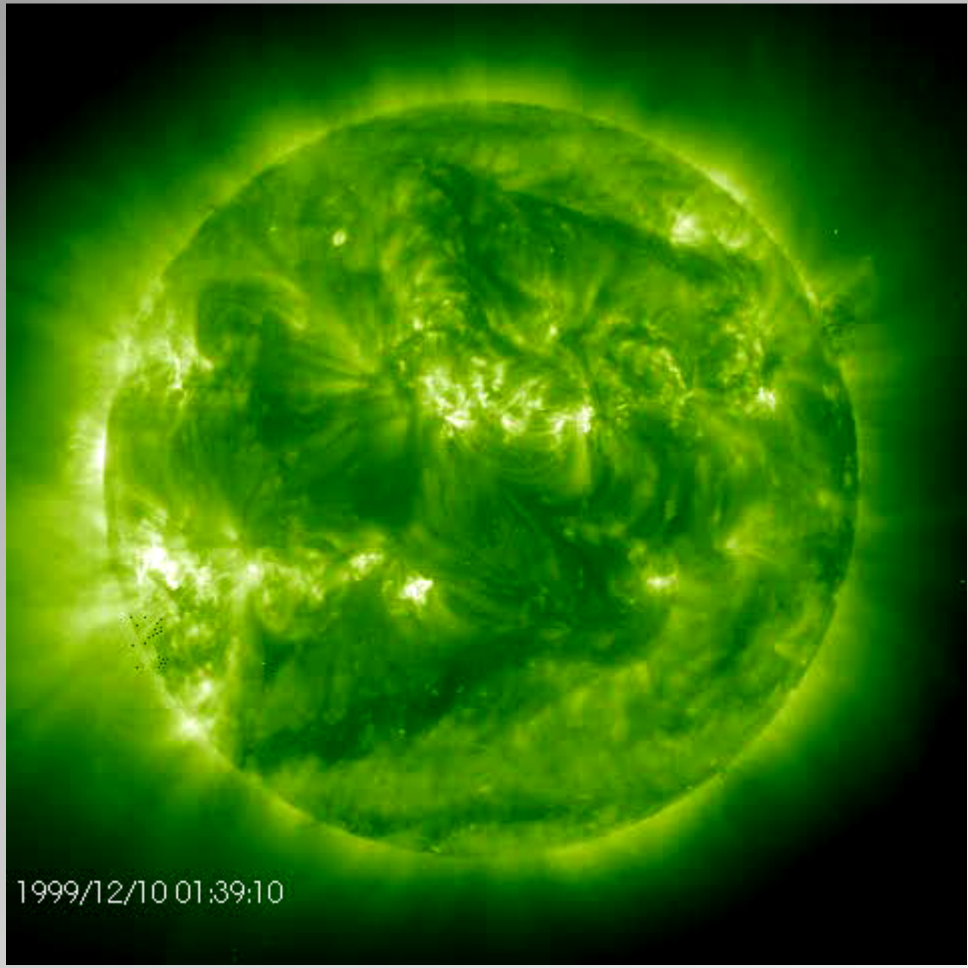
- Solar flare X-ray mission
- March 2001



Hinode/Solar-B

- Japan/UK/USA Mission
- Successor of Yohkoh
- September 2006

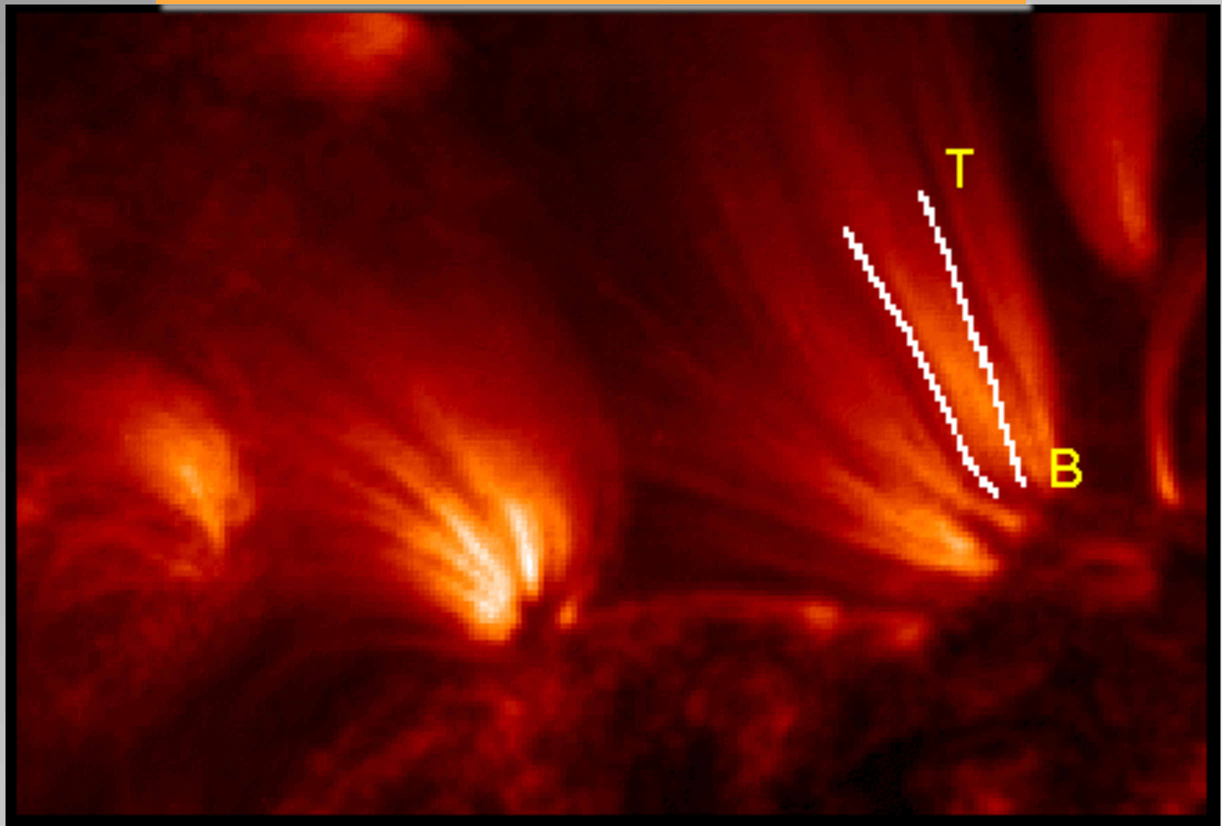
Do we see MHD waves?



• Dynamic and strongly inhomogeneous corona

Do we see MHD waves?

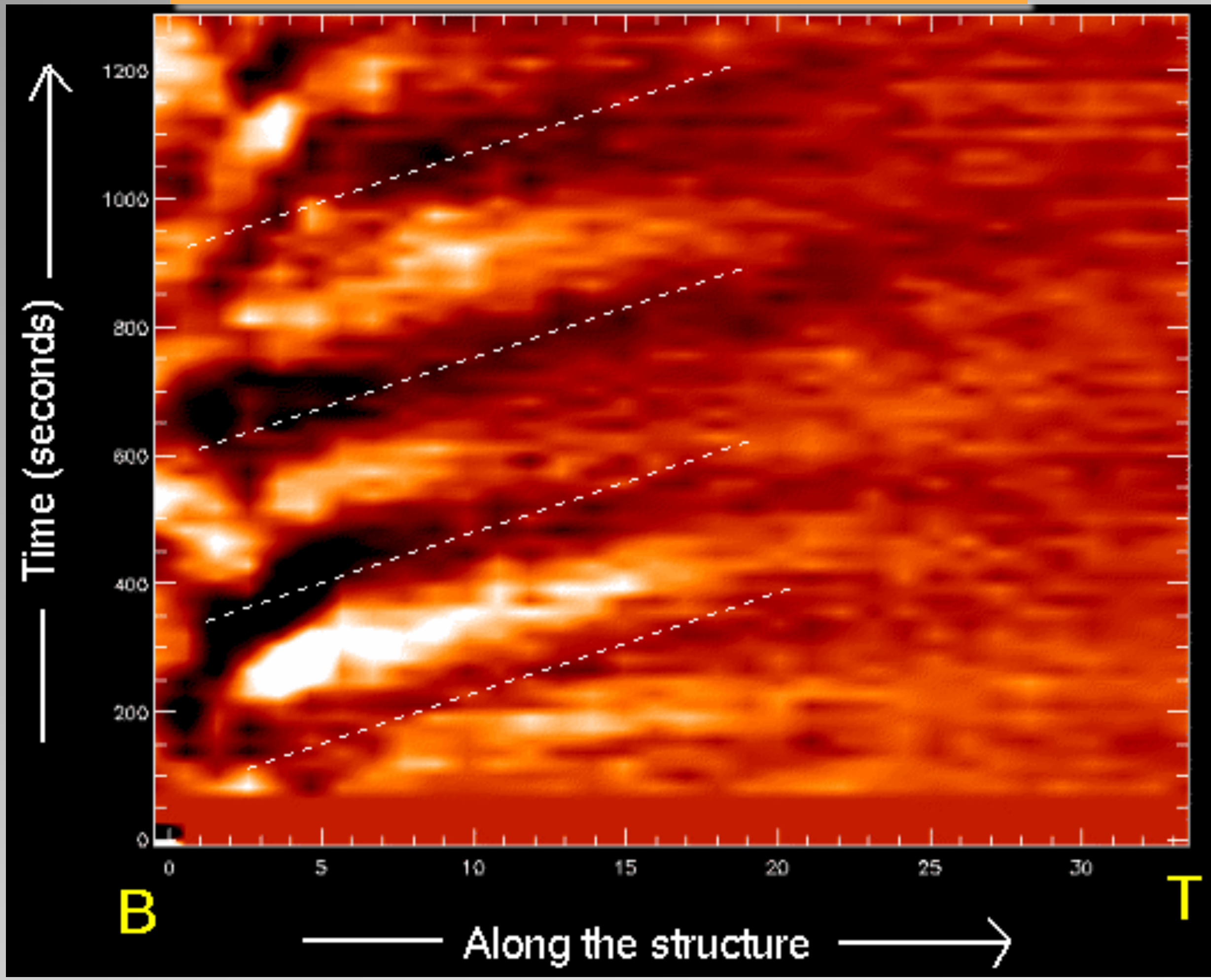
Surfing magnetic loops



- Example: Rapid (every 15 s) TRACE 171 Angstrom image
- Track changes in brightness
- Wave travels outwards from B to T

Do we see MHD waves?

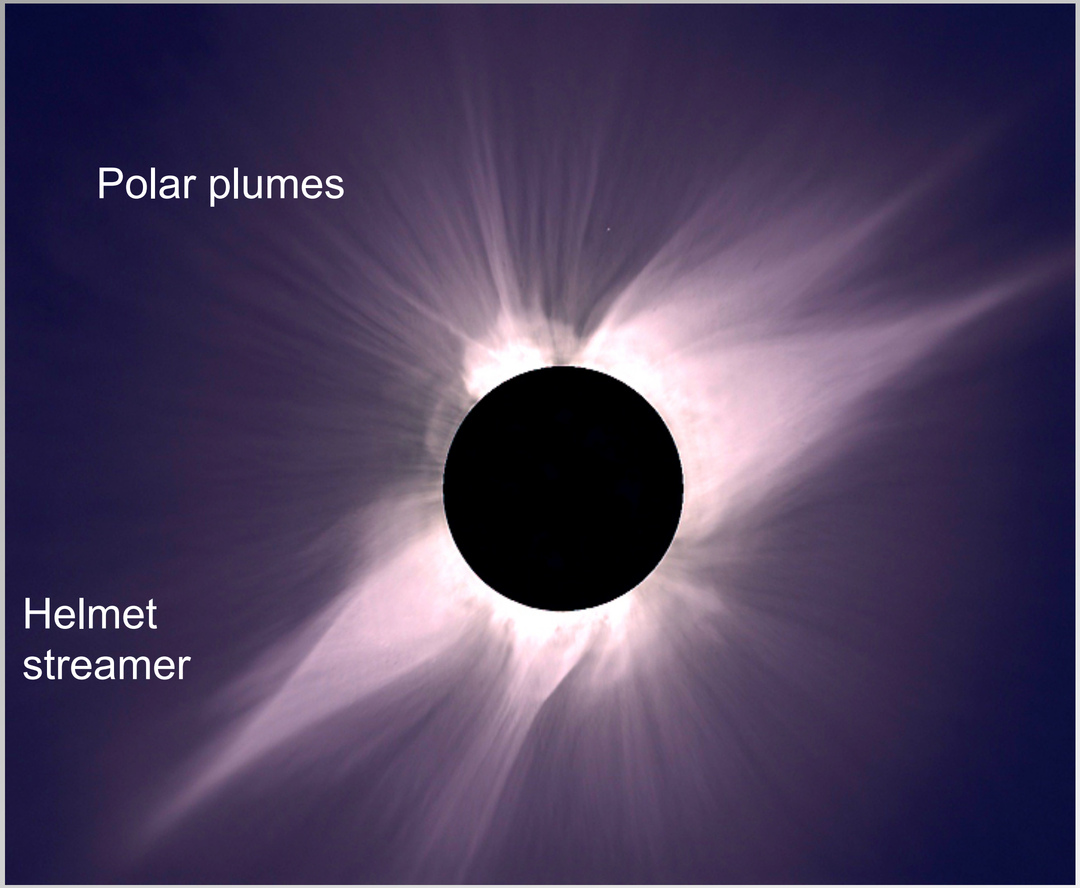
Surfing magnetic loops



• Difference image in brightness out from the loop base

Do we see MHD waves?

Compressive waves in solar plumes



Do we see MHD waves?

Compressive waves in solar plumes

DeForest & Gurman (1998) and Ofman et al. (1999) with SOHO/EIT and TRACE have detected and investigated wave motions in polar plumes. Main properties of these waves:

- Outwardly propagating perturbations of the intensity (plasma density) at 1.01-1.2 $R_{\odot}$,
- Quasiperiodic groups of 3-10 periods,
- Periods about 10-15 min,
- The duty cycle is roughly balanced,
- Speeds are about 75-150 km/s,
- Amplitude (in density) is about 2-4 % of the background and grows with height.

Ofman et al. (1997) using white light channel (WLC) of the SOHO/UVCS have detected density fluctuations in coronal holes with periods ≈ 9 min at 1.9 $R_{\odot}$.

Do we see MHD waves?

Compressive waves in long loops

Berghmans & Clette (1999), with SOHO/EIT have observed compressive propagating disturbances in coronal loops (on the disk). Main findings:

Upwardly propagating perturbations of the intensity (plasma density) (very similar to the plume case, but on the disc),

With speed about 65-165 km/s,

Amplitude is ~ 2% in intensity (~ 1 % in density),

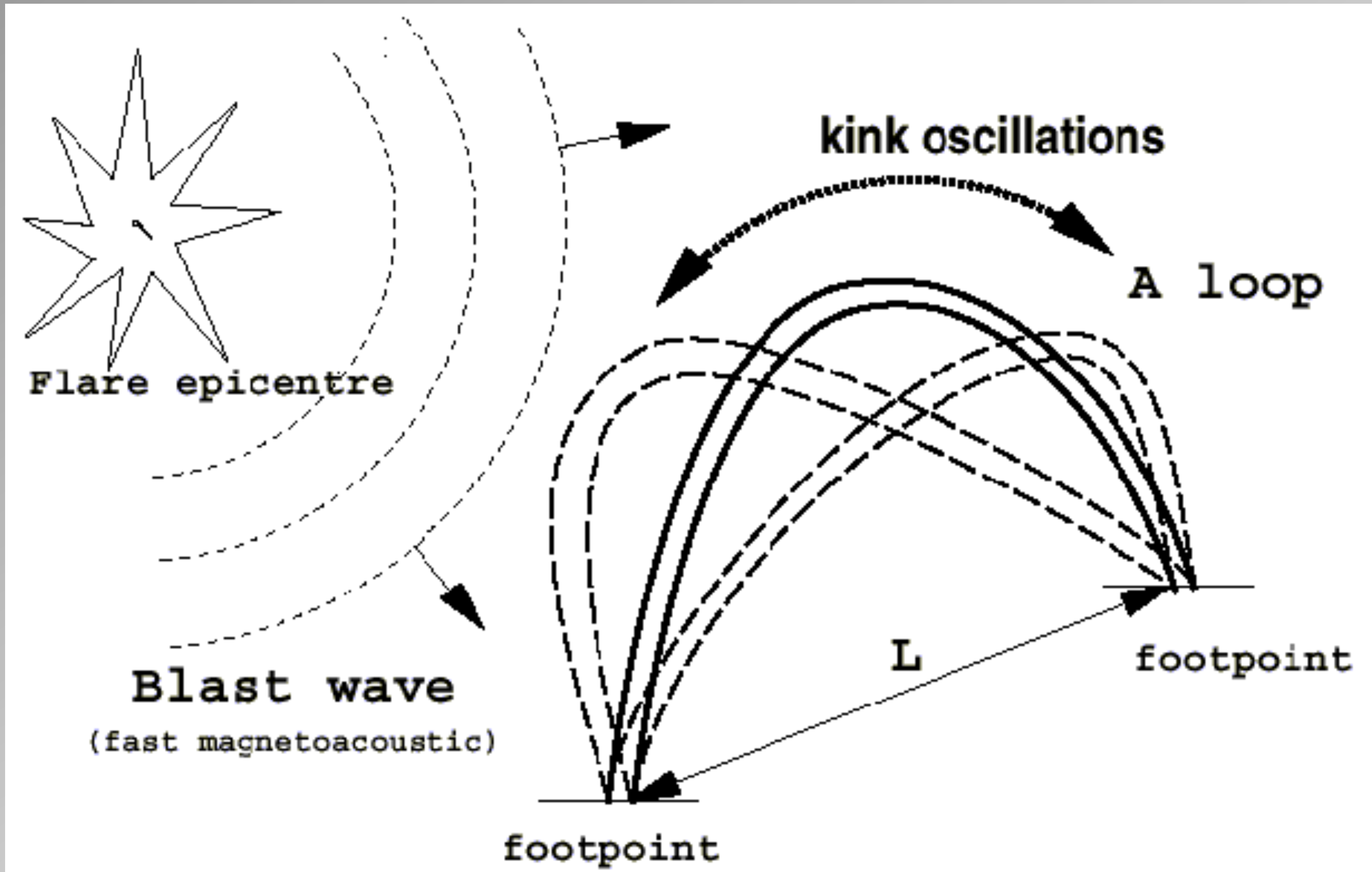
The height growth of the amplitude has not been found,

No manifestation of downward propagation.

Travelling along almost all loops analysed.

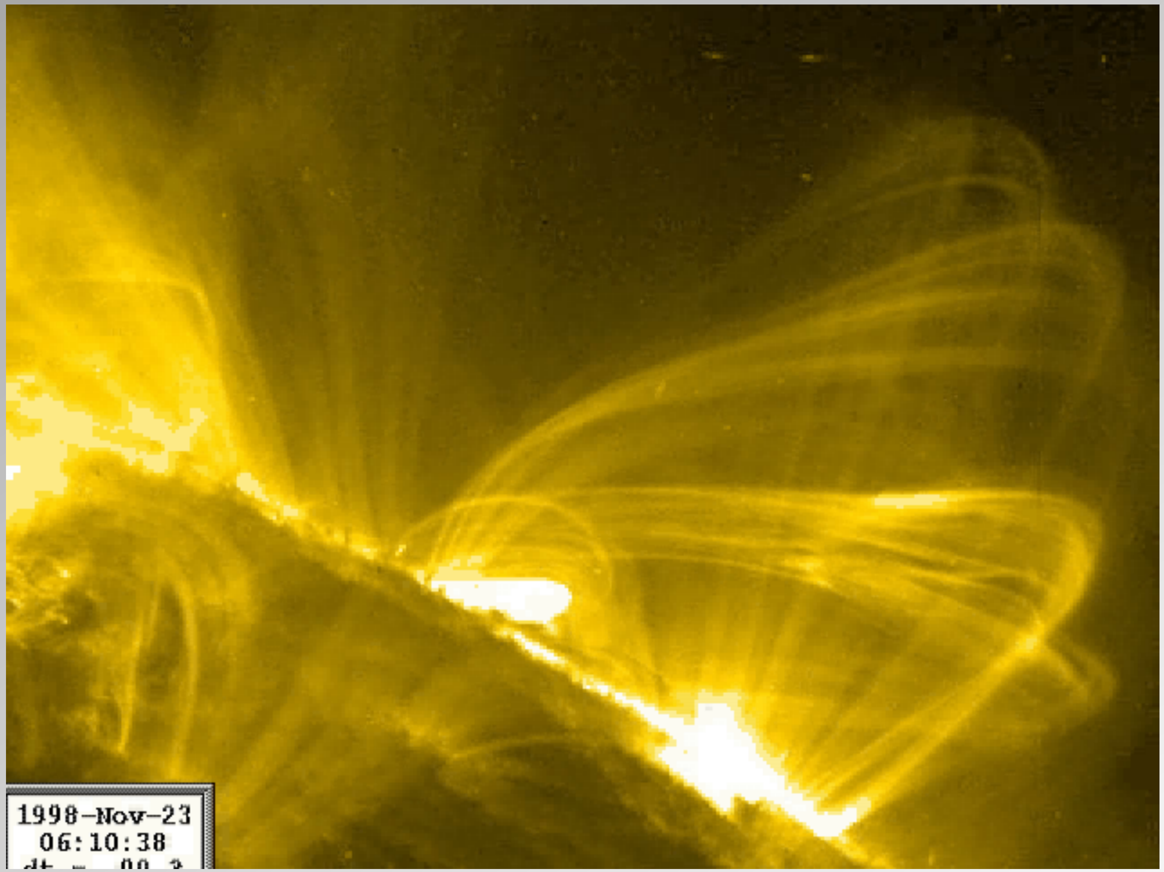
Similar waves are observed with TRACE.

Do we see MHD waves?



Do we see MHD waves?

SOHO/TRACE examples (mainly TR and higher)



Do we see MHD waves?

Post-flare loop oscillations

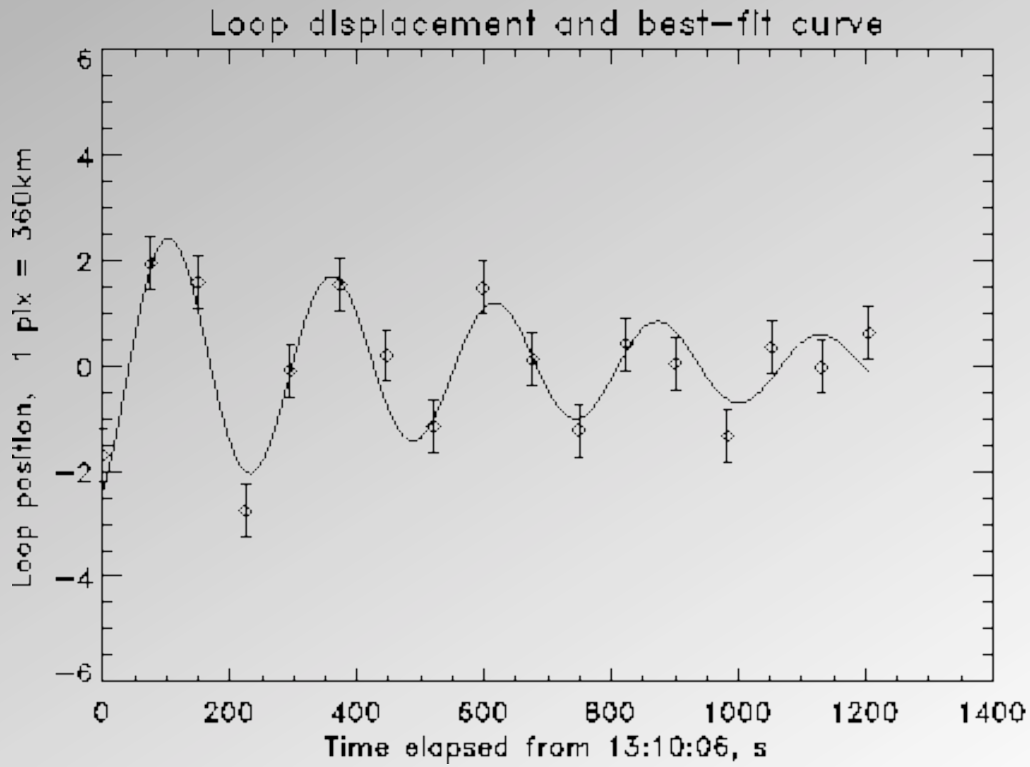
Aschwanden et al. (1999) and Nakariakov et al (1999) with TRACE have observed and investigated decaying kink-like oscillations of coronal loops, excited by a nearby flare (by a coronal Moreton wave?).

Main properties:

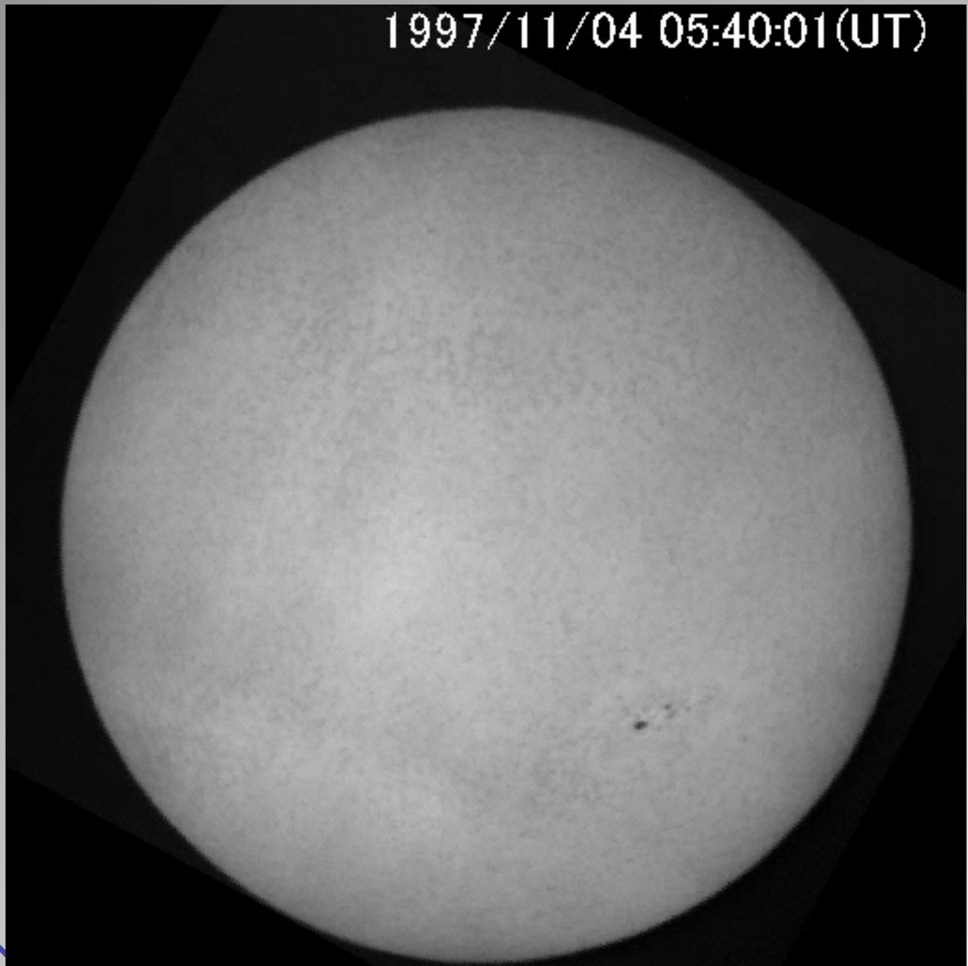
Oscillations are quasi-periodic with periods of several minutes (256 s),

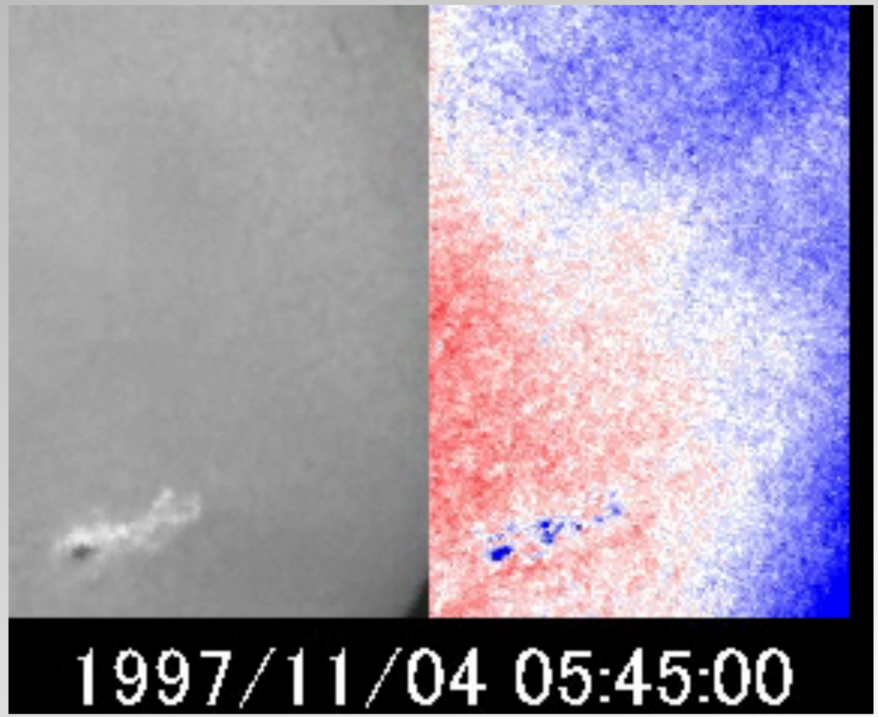
Initial displacement amplitudes are about several Mm for loop radii about 100 Mm,

Decay time about 14.5 min.



Do we see MHD waves? - Moreton waves





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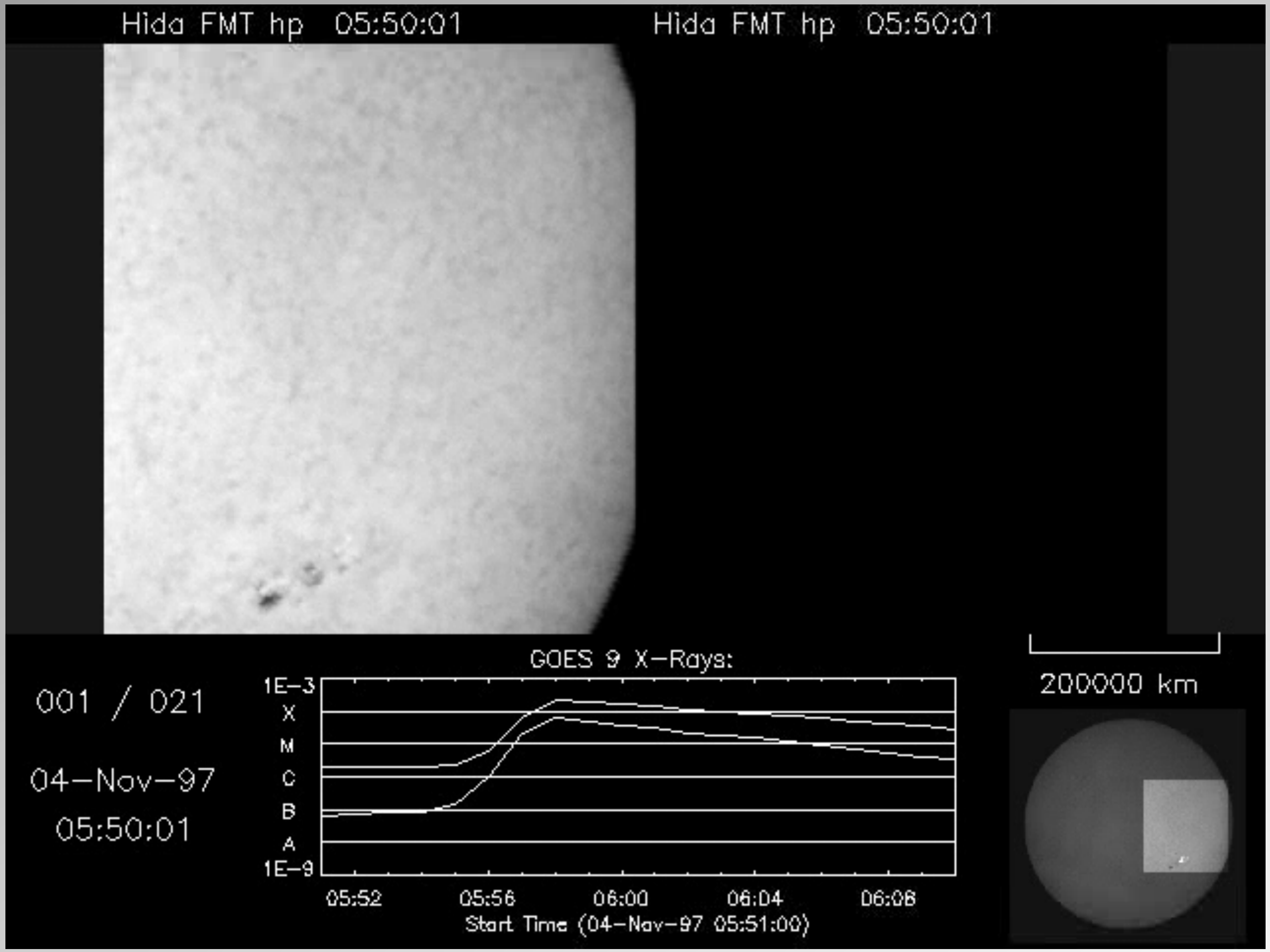


Do we see MHD waves?

New: Coronal Moreton waves

Thompson et al. (1999) with SOHO/EIT have investigated a global coronal wave generated by the coronal mass ejection or a flare and occupying a significant part of the solar disk. This wave has been called a *coronal Moreton wave*.

Properties accumulated from observations of more than 50 events:
(see ***http://umbra.nascom.nasa.gov/bjt/lscd/*** or Ballai et al. 2005 for details)

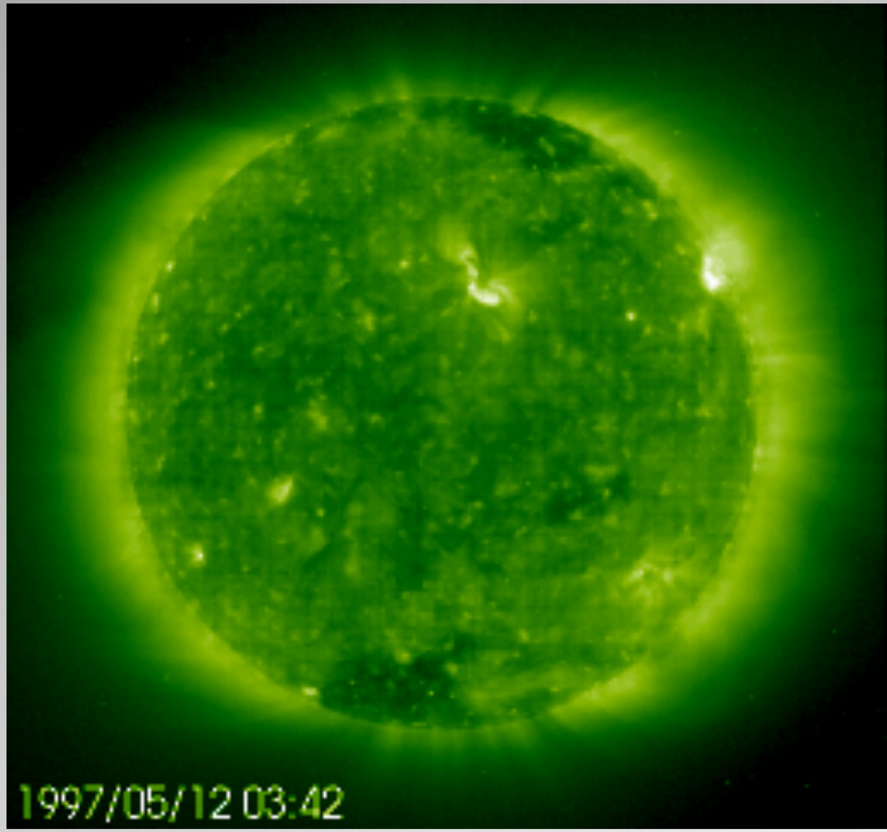
The waves prefer to propagate radially from the epicentre, stopping at neutral lines and coronal hole boundaries, and distorted by active regions.

Speeds range is from 200-600 km/sec.

Active regions distort the waves locally, bending them possibly toward the lower Alfvén speed regions.

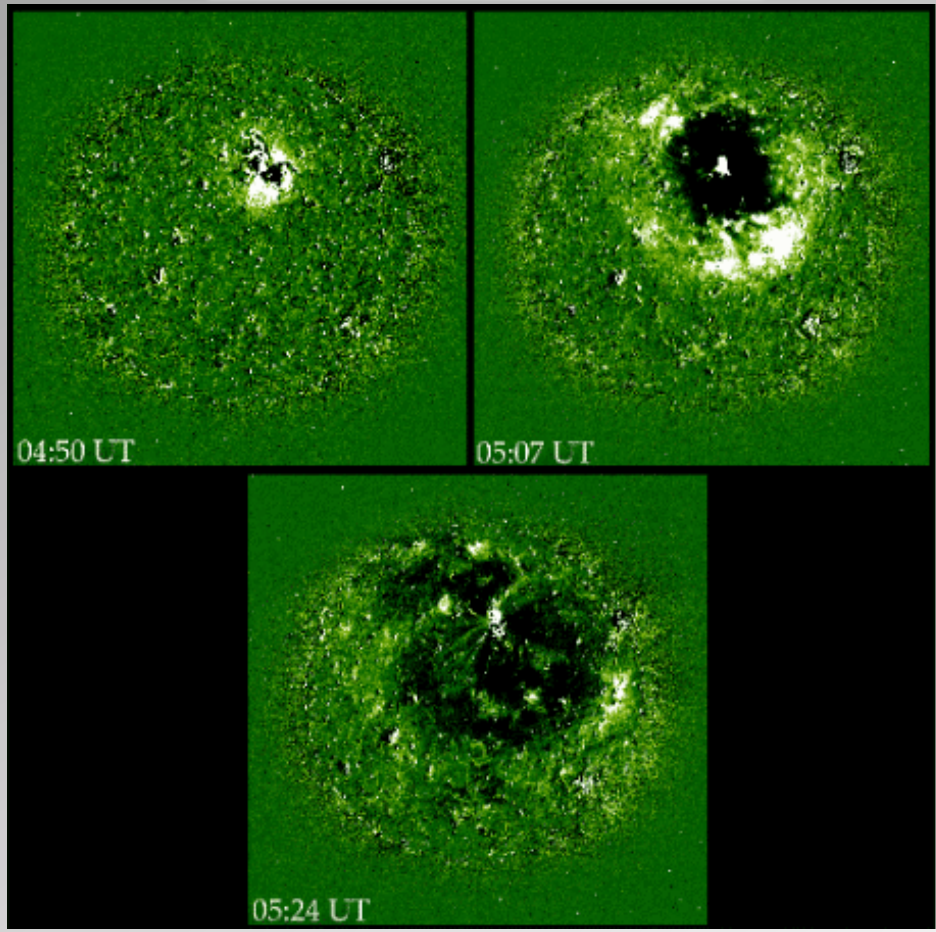
The waves can cause "visible deflection" of coronal magnetic field lines and probably are associated with filament oscillations.

Do we see MHD waves? – Coronal Moreton waves

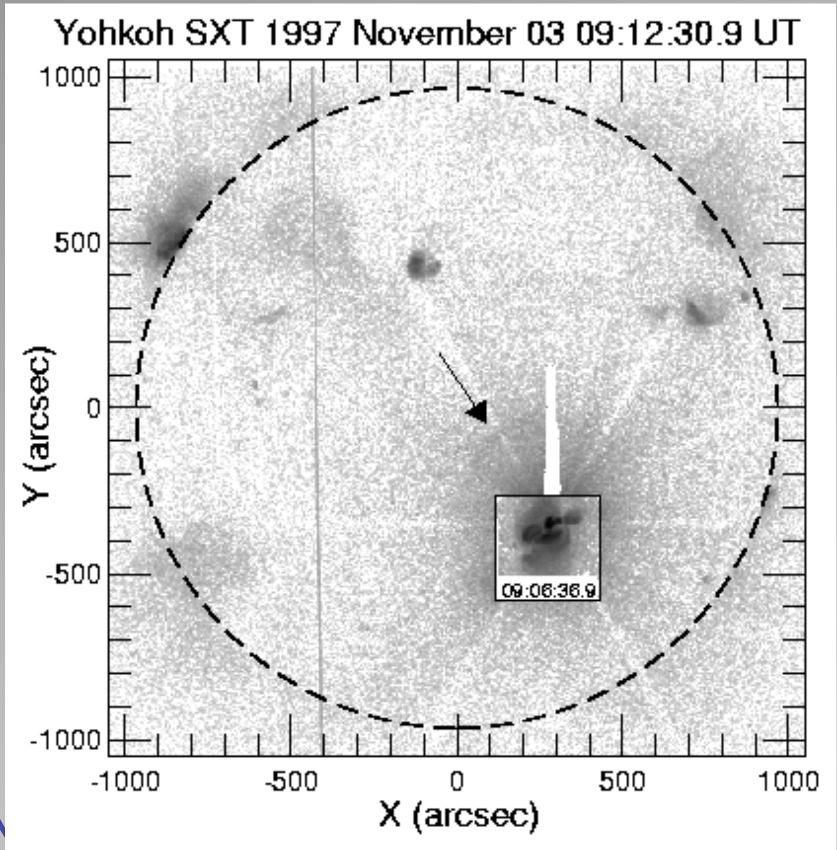


Do we see MHD waves? – Coronal Moreton waves

- Moreton waves on difference images after solar eruption



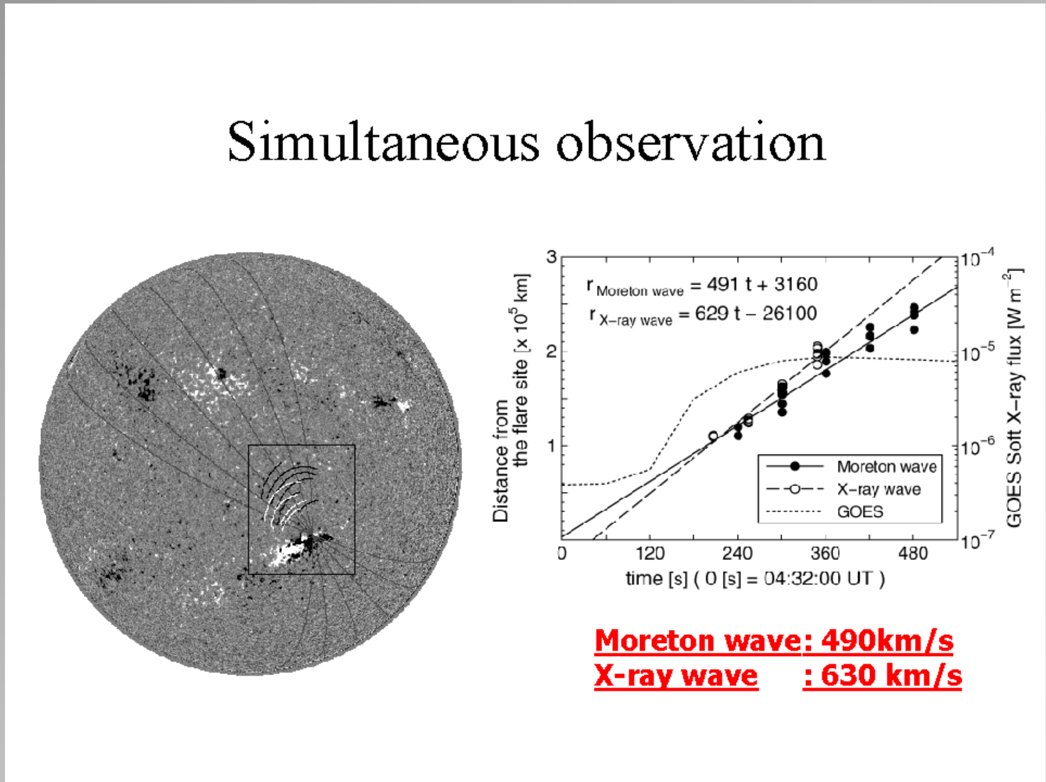
Do we see MHD waves? – X-ray waves



- Seen by Yohkoh/SXT propagating in the corona
- Interpreted as coronal MHD fast-mode weak shock (Narukage et al. ’02)
- Propagation speed of 630±100 km/s
- Believed to be the coronal counterpart of chromospheric Moreton waves (?)

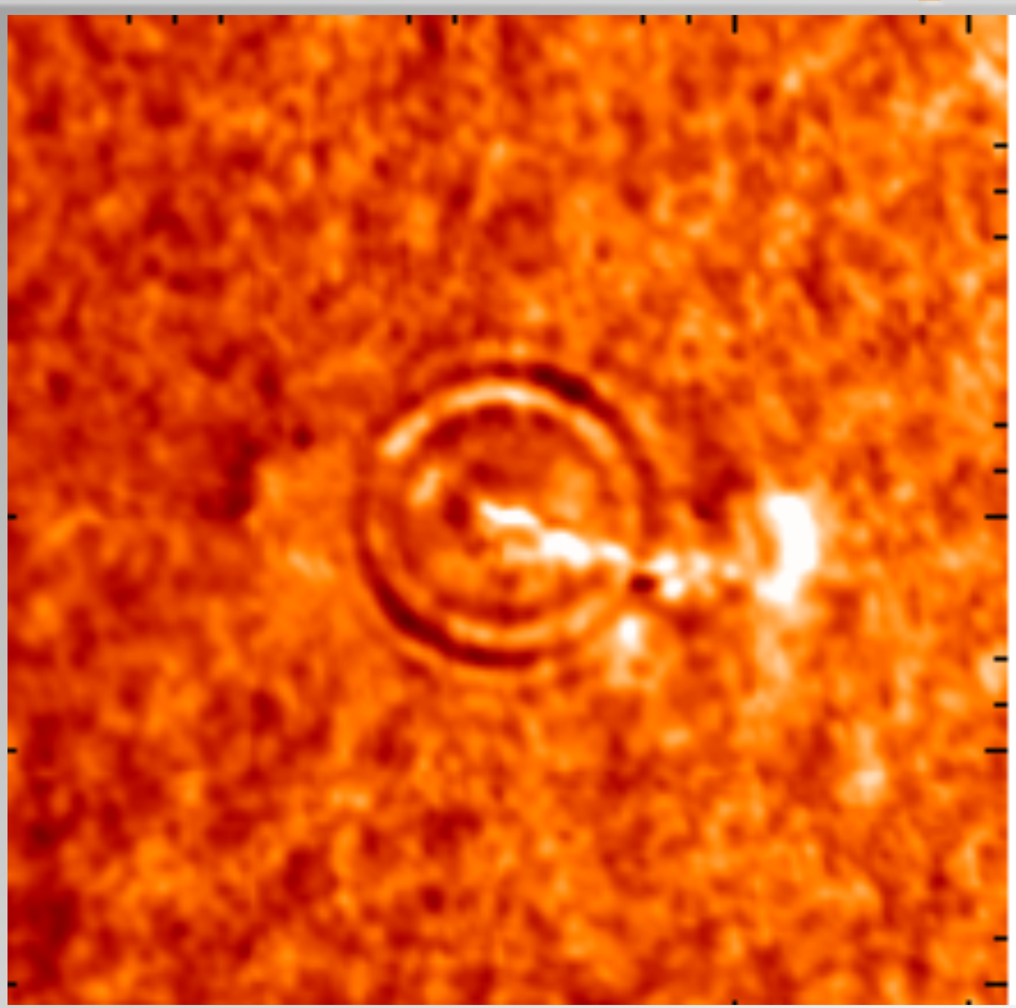
Do we see MHD waves?

Moreton waves ⇔ X-ray waves



- 1997.11.03 NOAA AR 8100
- Both propagate in the same direction and agree in location
- X-ray waves are well correlated to Moreton waves
- X-ray waves are the coronal counterpart of the Moreton waves (Narukage et al '02)

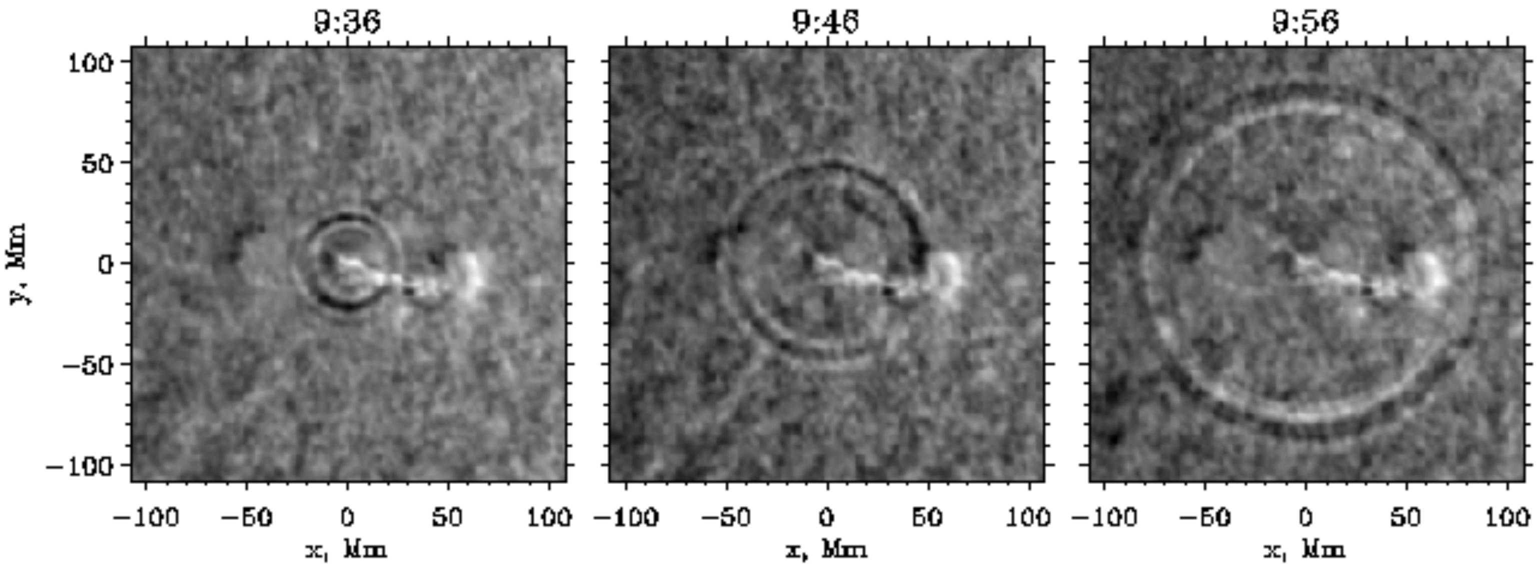
Do we see MHD waves? – Sun quakes



- Solar quakes: high energy electrons slam the solar surface

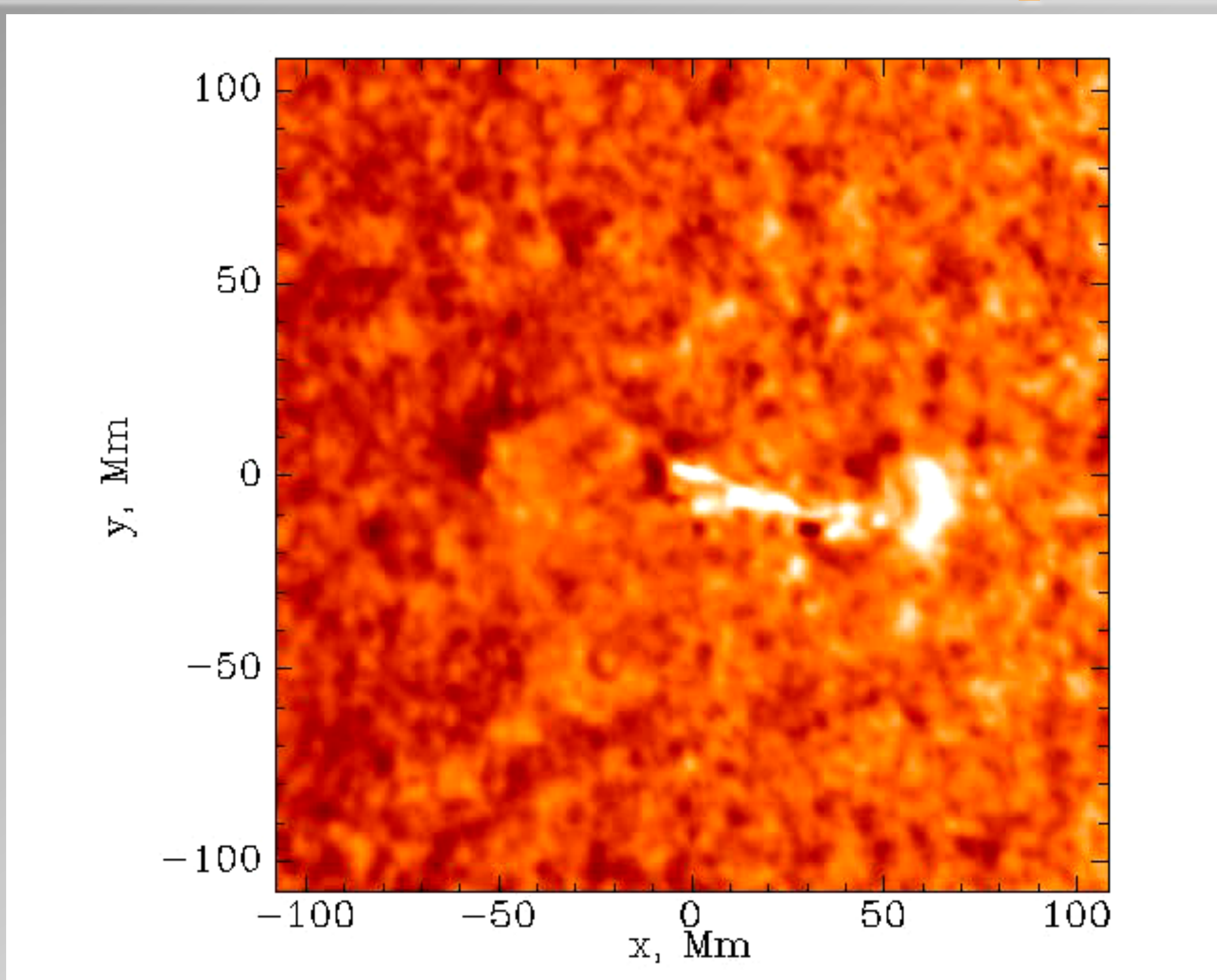
Do we see MHD waves? – Sun quakes

- Solar quakes: high energy electrons slam the solar surface

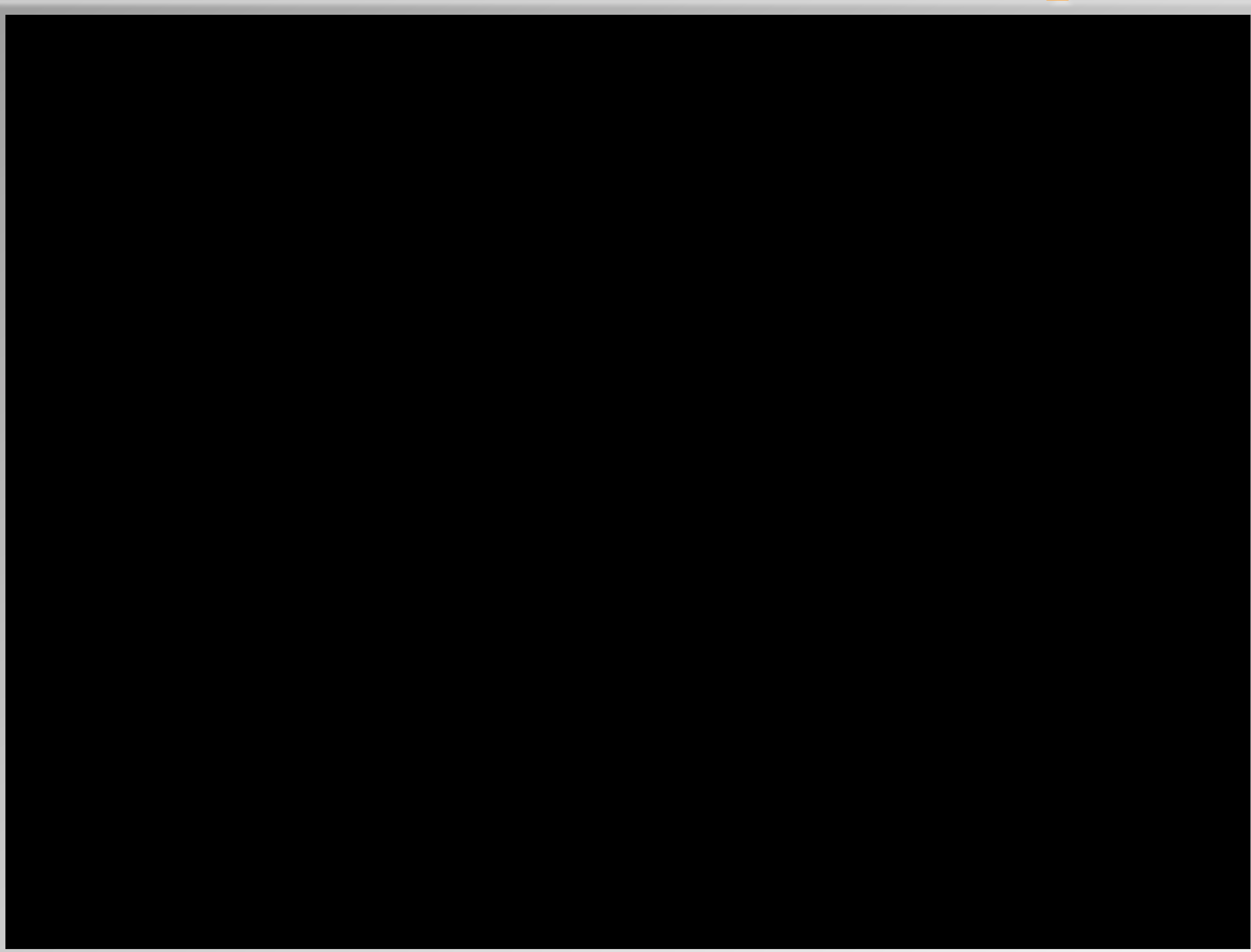


Do we see MHD waves? – Sun quakes

- Solar quakes: high energy electrons slam the solar surface



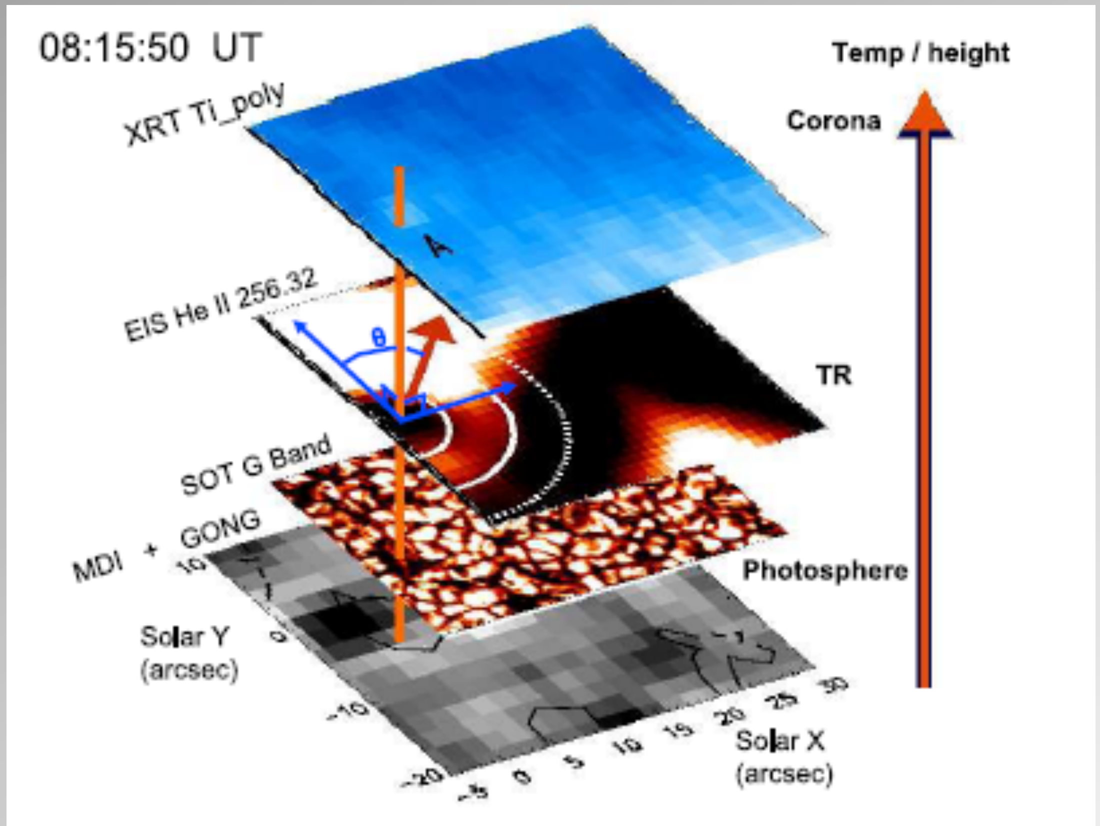
Do we see MHD waves? – TR quakes



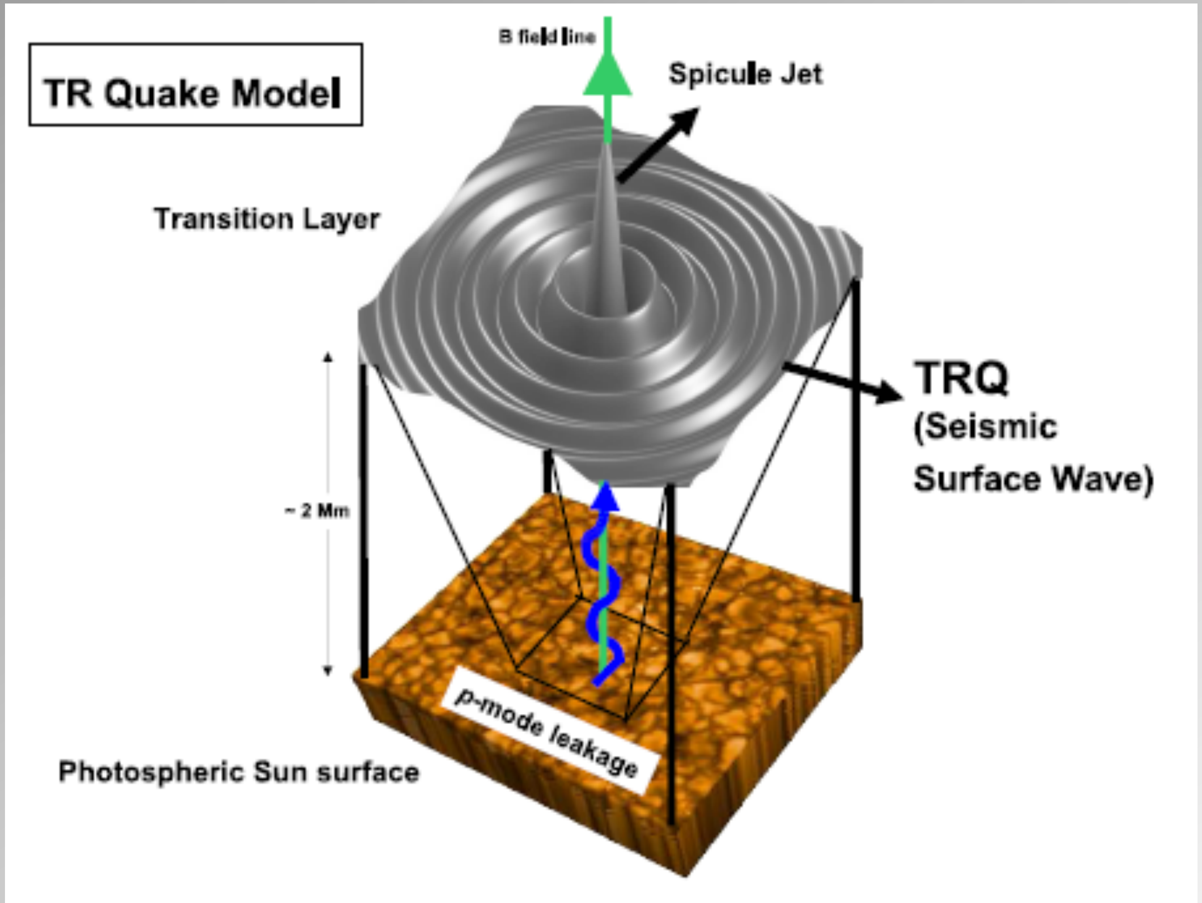
- Solar TR quakes: MAGSWs

Do we see MHD waves? – TR quakes

- Solar TR quakes: MAGSWs



Do we see MHD waves? – TR quakes



- Solar TR quakes: MAGSWs

Do we see MHD waves?

Non-thermal broadening of coronal emission lines

[Most probably associated with MHD waves).]

Measured broadening of minor ion spectral lines is formed by two effects, thermal broadening and non-thermal broadening associated with the Doppler shift due to unresolved line-of-sight motions

$$T_{\mathrm{eff}} = T_{\mathrm{i}} + \alpha \frac{m_{\mathrm{i}}}{2k} < v_{\mathrm{LOS}}^2 >,$$

where T_{i} is the temperature of the line forming ion, k is the Boltzmann constant , v_{LOS} is the line-of-sight (LOS) velocity, $2/3<\alpha<1$

Non-thermal broadening of the UV and EUV coronal lines has been known for 25 years from *Skylab*.

Do we see MHD waves?

Non-thermal broadening of coronal emission lines (ctd)

Recent findings:

Ofman & Davila (1997) using SOHO/UVCS measured unresolved motions with speeds up to 300 km/s at about 1.7 $R_\odot$.

Erdélyi et al. (1998) using SOHO/SUMER found that the non-thermal center-to-limb LOS velocity increases from few km/s to almost 100 km/s in coronal loops.

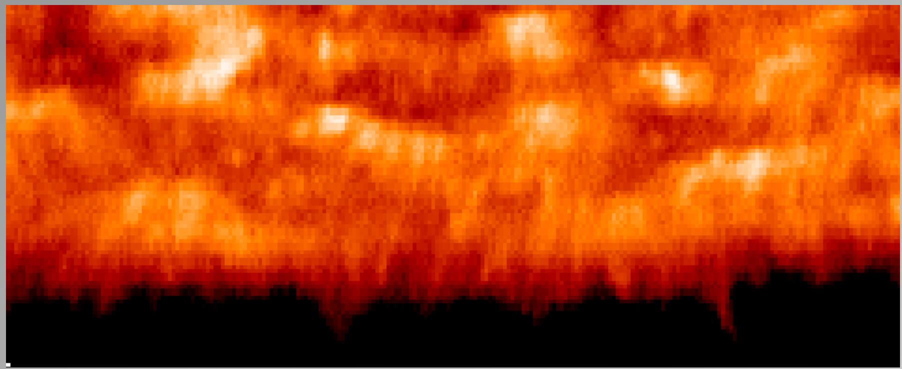
Banerjee et al. (1998) using SOHO/SUMER found that the non-thermal LOS velocity increases from 27 km/s at 20 Mm above the limb to 46 km/s at 62 Mm.

Chae et al. (1998), SOHO/SUMER, LOS velocities of 20-30 km/s on the disc

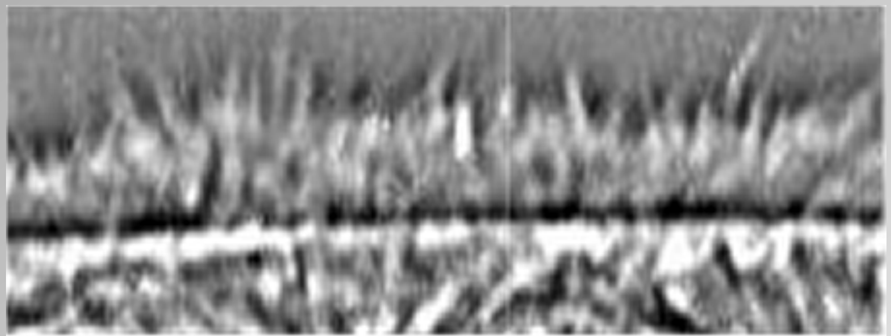
Esser et al. (1999), SOHO/UVCS, LOS velocities of 20-23 km/s at 1.35-2.1 $R_\odot$

There is some discrepancy in results found using different instruments. However, the results clearly show the presence of the unresolved line-of-sight plasma motions caused by waves and/or turbulence in the corona.

Do we see MHD waves?- Solar spicules



SOHO Image of the Solar limb taken March '96



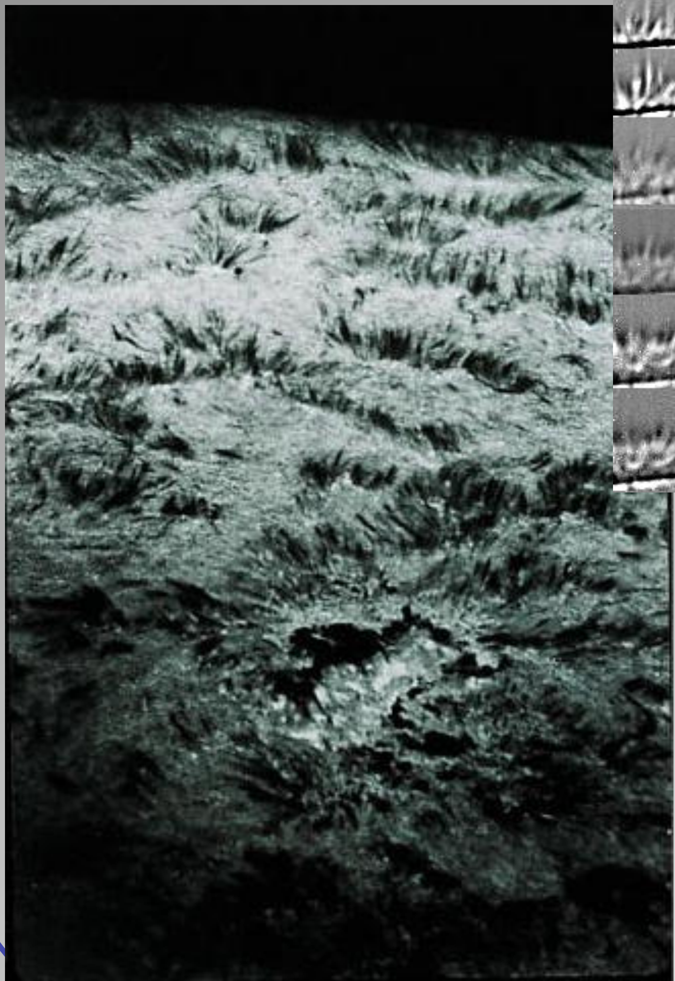
Hα Image from the Big Bear Solar Observatory, California

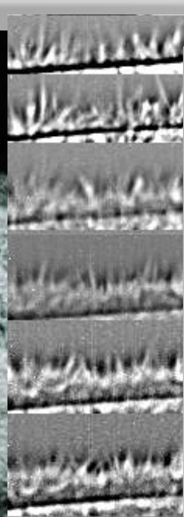
- Solar spicules are thin, hair-like jets of gas seen on the solar limb in chromospheric emission lines
- They occur predominantly at supergranule boundaries and appear to be guided along the intense magnetic flux tubes gathered there
- Typical properties are:

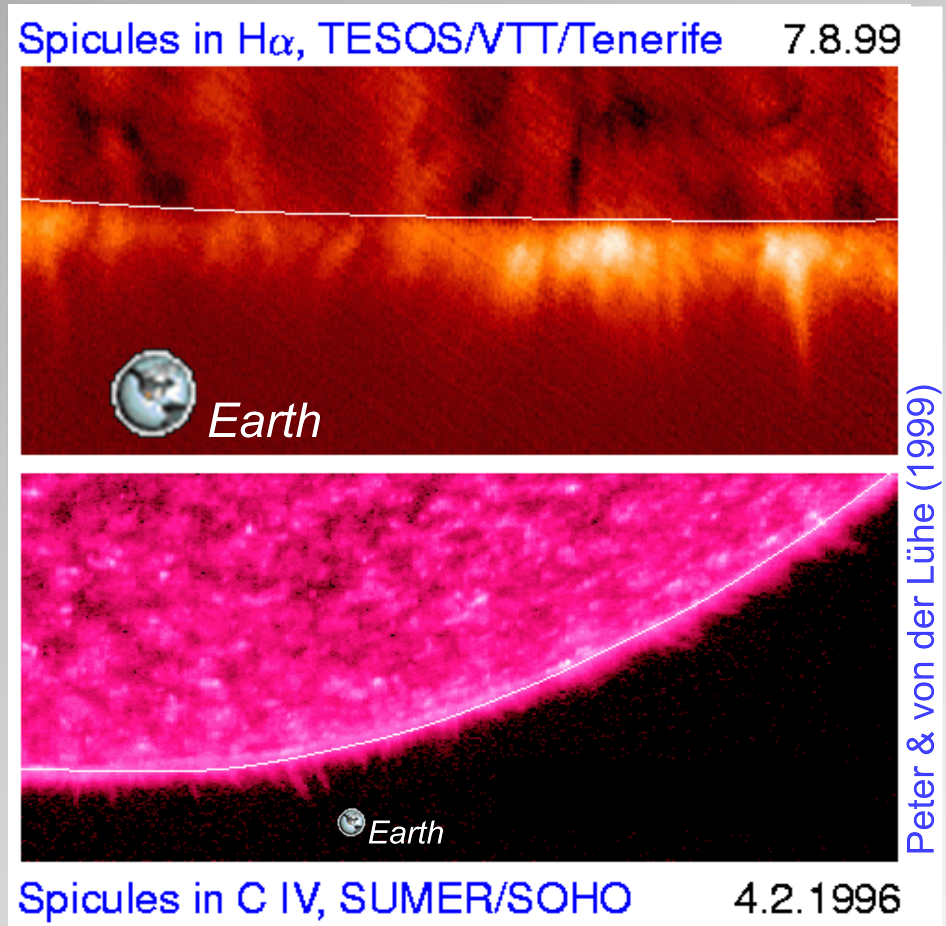
Width	200-1000km
Height	5000-10000km
Lifetime	5-15mins
Axial Velocity	$20 kms^{-1}$
Temperature	5000-15000K
Density	$0.5\text{-}2.5 kgm^{-3}$

- Some spicules display rapid rotation about their axis, typically of the order of $25km\ s^{-1}$
- The spicule rise is probably not ballistic, although the evidence for this is not conclusive

Do we see MHD waves?- Solar spicules







Do we see MHD waves?

Solar tornadoes

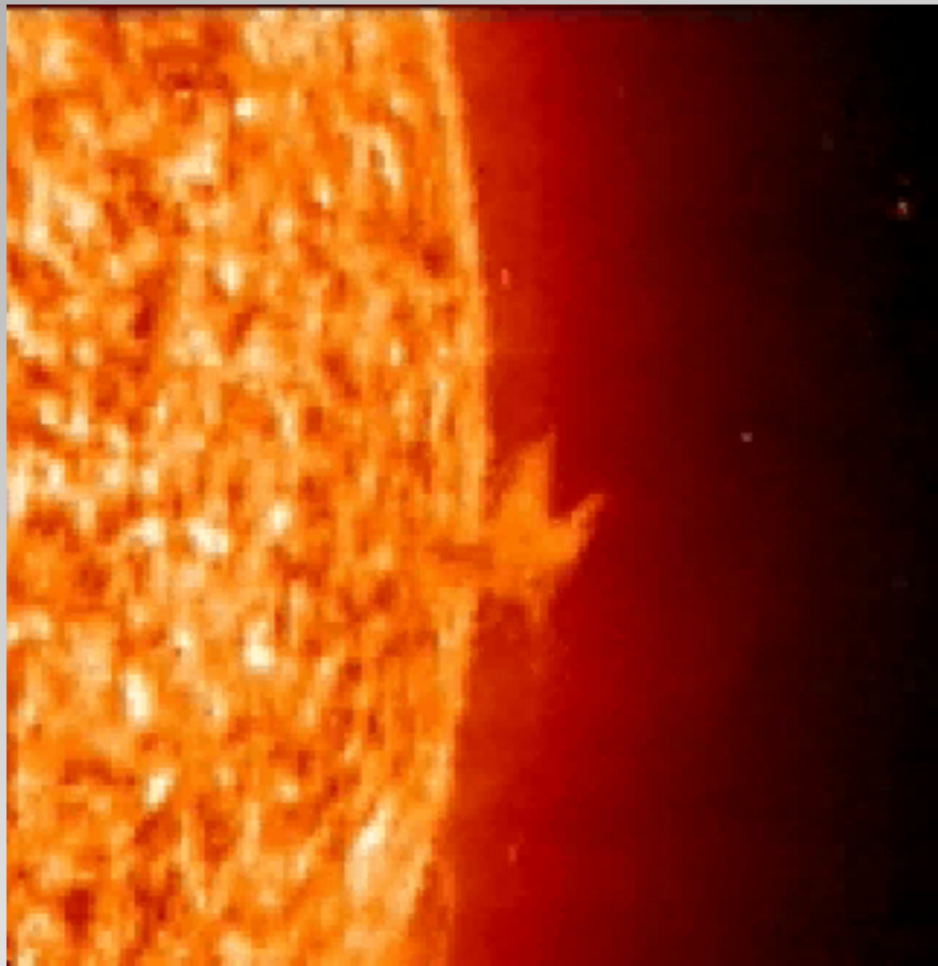
(May be connected with MHD waves).

Pike & Mason (1998) with SOHO/CDS:

Macrospicule-like (a jet) features have been identified in the polar regions both on the limb and disk.

Blue- and red-Doppler-shifted emission occur on either side of the feature axis, indicating the presence of rotation (called *solar tornado*).

The rotation velocities increase with height.



Atmospheric seismology

Oscillations ubiquitous in Sun

Solar interior

- Global oscillations
- p/f/g-modes

Solar atmosphere

- More local oscillations
- Sunspot oscillations, prominence oscillations, coronal loop oscillations, plume oscillations
- EIT waves?

Unifying feature of variety of solar atmospheric oscillations

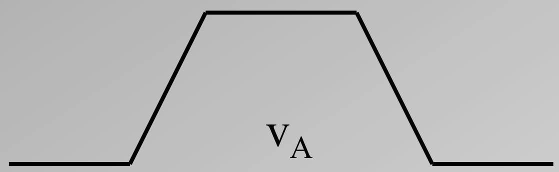
- Waveguide concept
- MHD description

Atmospheric seismology

Oscillations ubiquitous in solar atmosphere

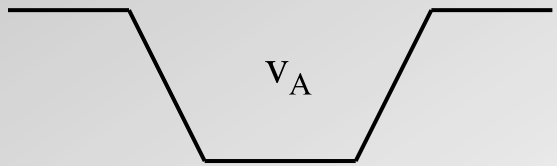
Lower atmosphere

- Ph, Ch, possibly TR
- Isolated flux tubes
- Effect of stratification



Higher atmosphere

- TR, corona
- Magnetic environment



Stratification leads to the Klein-Gordon effect

Roberts (1981), Rae & Roberts (1982)

Atmospheric seismology

What is the motivation?

- Source of atmospheric heating; solar wind/particle acceleration
- Understand atmospheric structures (spicules, prominences, loops, plumes, etc.)

Observations

spectroscopic

imaging

Wave properties (speed, amplitude, spectrum)

Atmospheric diagnostic parameters (temperature, density)

Geometric properties of waveguides (structuring, shape, curvature)

Atmospheric physical parameters (B, fine structure, transport coefficients)

Coronal seismology born (Roberts et al. 1984)

Linear theory of MHD waves

- Static/steady stationary background
- Superimpose linear motions on this background
- Write physical quantities as

$$f(r,t)=f_0(r)+f_1(r,t); \quad |f_1|/|f_0| \ll 1$$

- Reduce full set of nonlin PDEs of MHD to a set of ODEs
- Choice: initial value problem, boundary value problem, eigenvalue problem
- Eigenvalue problem of linear waves/oscillations: $\exp(i\omega t)$

Linear theory of MHD waves

Linearised ideal MHD equations

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \mathbf{V}_1 = 0$$

$$\rho_0 \frac{\partial \mathbf{V}_1}{\partial t} = -\nabla\left(p_1 + \frac{\mathbf{B}_0 \cdot \mathbf{B}_1}{\mu} \right) + \frac{(\mathbf{B}_0 \cdot \nabla)\mathbf{B}_1}{\mu}$$

$$\frac{\partial \mathbf{B}_1}{\partial t} = \nabla \times (\mathbf{V}_1 \times \mathbf{B}_0)$$

$$p_1 = c_s^2 \rho_1, \qquad c_s^2 = \gamma p_0 / \rho_0$$

Linear MHD waves in uniform plasma

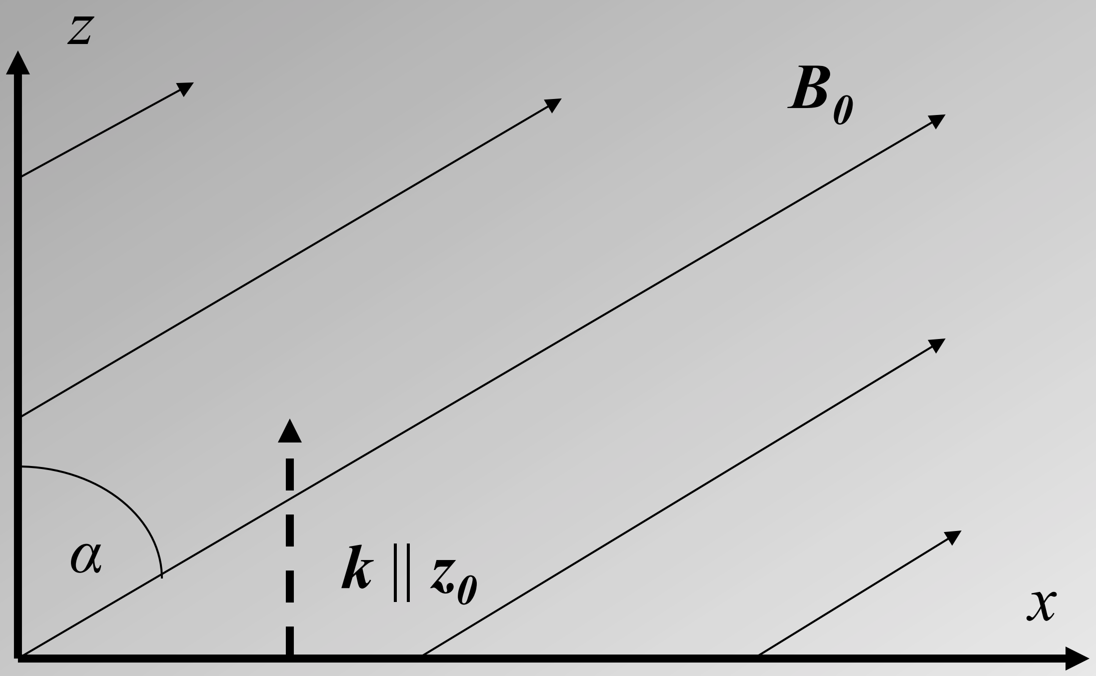
- No characteristic length scale defined by the equilibrium
- Constant equilibrium magnetic field , e.g.

$$\mathbf{B_0} = B_0 \sin\alpha \mathbf{x_0} + B_0 \cos\alpha \mathbf{y_0}$$

- Superposition of linear waves

exp(i$k_x x$ + i$k_y y$ + i$k_z z$), $\boldsymbol{k}$=(k_x, k_y, k_z)=wave vector

Linear MHD waves in uniform plasma



Characteristic speeds:

Alfvén speed $v_A = B_0(4\pi\rho_0)^{-1/2}$

sound speed $c_s = (\gamma p_0 / \rho_0)^{1/2}$

Linear MHD waves in uniform plasma

Consider dynamics of perturbations of this stationary state. In the linear limit, the set of MHD equation splits into two uncoupled subsets:

Linear MHD waves in uniform plasma

Consider dynamics of perturbations of this stationary state. In the linear limit, the set of MHD equation splits into two uncoupled subsets:

(i) for the variables V_y and B_y (Alfvén wave)

(ii) and for ρ, p, V_x, V_z and B_x (magnetoacoustic waves)

Linear MHD waves in uniform plasma

Alfvén waves

$$\left(\frac{\partial^2}{\partial t^2} - v_{Az}^2 \frac{\partial^2}{\partial z^2}\right) V_y = 0 \qquad v_{Az} = B_0 \cos\alpha / (4\pi\rho_0)^{1/2}$$

Properties:

(i) Transverse oscillation driven by [redacted]

(ii) [redacted] perturb density ➔ [redacted]

(iii) [redacted] propagate across field lines

(iv) Group velocity ($\delta\omega/\delta k$) is along B_0

Linear MHD waves in uniform plasma

Alfvén waves

$$\left(\frac{\partial^2}{\partial t^2} - v_{Az}^2 \frac{\partial^2}{\partial z^2}\right) V_y = 0 \qquad v_{Az} = B_0 \cos\alpha / (4\pi\rho_0)^{1/2}$$

Properties:

(i) Transverse oscillation driven by magnetic tension forces

(ii) perturb density ➔

(iii) propagate across field lines

(iv) Group velocity ($\delta\omega/\delta k$) is along B_0

Linear MHD waves in uniform plasma

Alfvén waves

$$\left(\frac{\partial^2}{\partial t^2} - v_{Az}^2 \frac{\partial^2}{\partial z^2}\right) V_y = 0 \qquad v_{Az} = B_0 \cos\alpha / (4\pi\rho_0)^{1/2}$$

Properties:

(i) Transverse oscillation driven by magnetic tension forces

(ii) Does not perturb density ➔ incompressible (in linear limit)

(iii) propagate across field lines

(iv) Group velocity ($\delta\omega/\delta k$) is along B_0

Linear MHD waves in uniform plasma

Alfvén waves

$$\left(\frac{\partial^2}{\partial t^2} - v_{Az}^2 \frac{\partial^2}{\partial z^2}\right) V_y = 0 \qquad v_{Az} = B_0 \cos\alpha / (4\pi\rho_0)^{1/2}$$

Properties:

(i) Transverse oscillation driven by magnetic tension forces

(ii) Does not perturb density ➔ incompressible (in linear limit)

(iii) Can't propagate across field lines

(iv) Group velocity ($\delta\omega/\delta k$) is along B_0

Linear MHD waves in uniform plasma

Alfvén waves

When $\boldsymbol{B_0} || z_0$ there can be two *linearly polarized* plane Alfvén waves, one perturbing V_y, B_y and the other V_x, B_x.

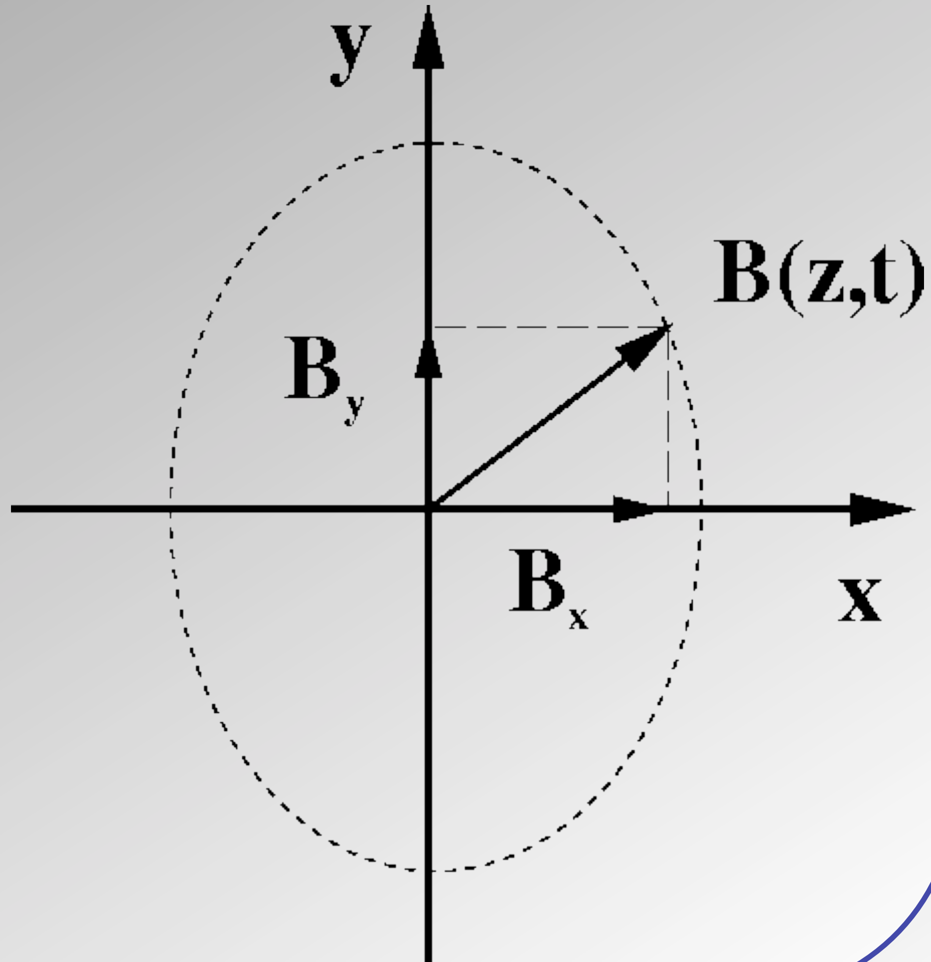
For harmonic perturbations [exp(iωt -kz)] combination of two linearly polarized waves gives us *elliptically polarized* Alfvén waves:

$$B_y = A\cos(\omega t - kz), \qquad A, B = \text{const}$$
$$B_x = B\sin(\omega t - kz),$$

Linear MHD waves in uniform plasma

Alfvén waves

The vector of the magnetic field perturbation rotates along an ellipse at the *x*,*y*-plane.



When $A = B$, the wave is *circularly polarized*, with $|\mathbf{B}|$=const.

Circularly polarized Alfvén waves (even of finite amplitude) are an *exact* solution of the ideal MHD equations for a uniform plasma

Linear MHD waves in uniform plasma

Magnetoacoustic waves

$$\left[\left(\frac{\partial^2}{\partial t^2} - v_{Az}^2 \frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial^2}{\partial t^2} - v_s^2 \frac{\partial^2}{\partial z^2}\right) - v_{Ax}^2 \frac{\partial^4}{\partial t^2 \partial z^2}\right] V_z = 0, \qquad v_{Ax} = B_0 \sin\alpha / (4\pi\rho_0)^{1/2}$$

Harmonic perturbations: $V_z \sim \exp[i(\omega t - kz)]$

Dispersion relation for MAW:

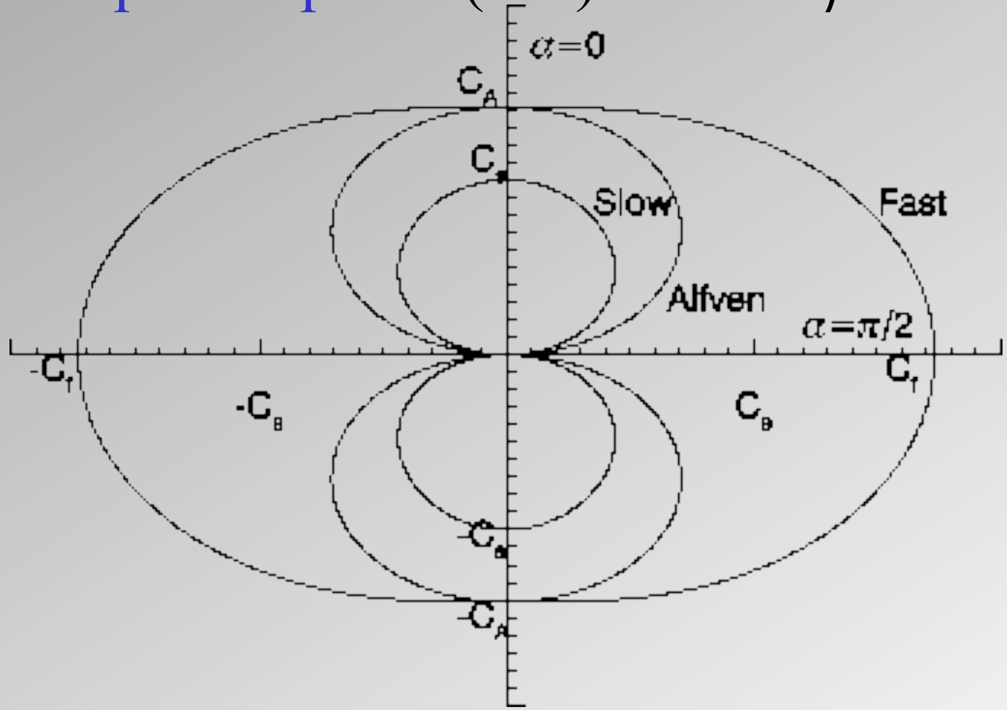
$$(\omega^2 - v_A^2 \cos^2 \alpha k^2)(\omega^2 - v_s^2 k^2) - v_A^2 \sin^2 \alpha \omega^2 k^2 = 0$$

DR bi-quadratic ➔ slow and fast magnetoacoustic waves

Linear MHD waves in uniform plasma

Magnetoacoustic waves

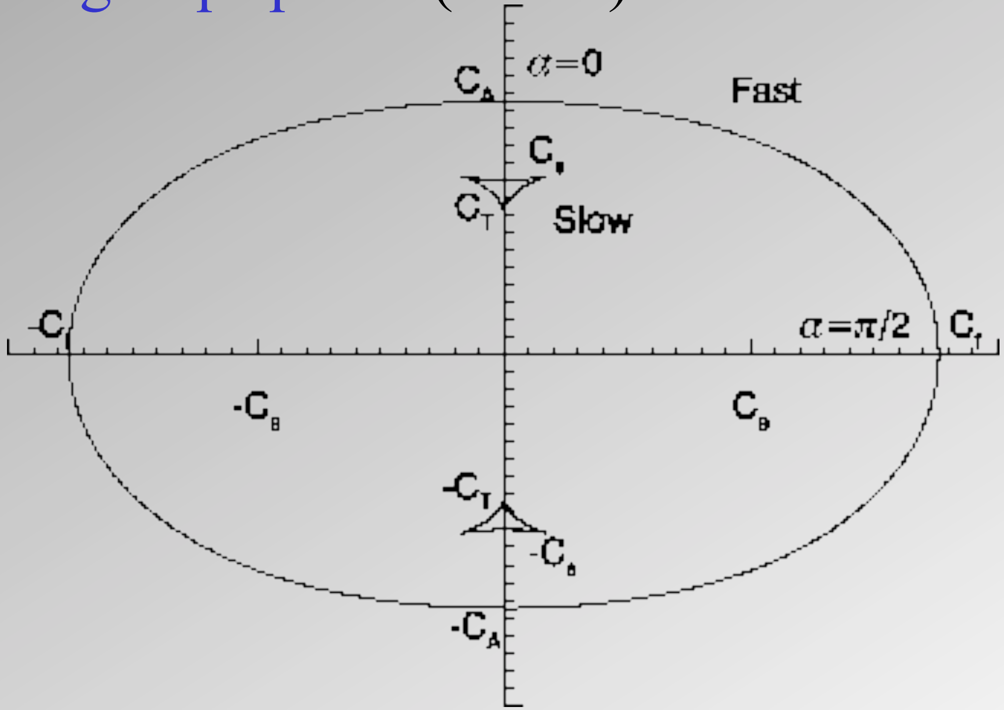
The polar plot for phase speeds (ω/k) for case β<1



Linear MHD waves in uniform plasma

Magnetoacoustic waves

The polar plot for group speeds ($d\omega/dk$)



Linear MHD waves in uniform plasma

Slow waves

Properties:

(i) Anisotropic wave propagation largely confined to magnetic field

(ii) Driven by

(iii) Does perturb density/pressure

(iv) propagate across field lines

Linear MHD waves in uniform plasma

Slow waves

Properties:

(i) Anisotropic wave propagation largely confined to magnetic field

(ii) Driven by magnetic pressure *and* tension forces

(iii) Does perturb density/pressure

(iv) propagate across field lines

Linear MHD waves in uniform plasma

Slow waves

Properties:

(i) Anisotropic wave propagation largely confined to magnetic field

(ii) Driven by magnetic pressure *and* tension forces

(iii) Does perturb density/pressure

(iv) Can’t propagate across field lines

Linear MHD waves in uniform plasma

Fast waves

Properties:

(i) Roughly isotropic wave propagation

(ii) Driven by magnetic pressure *and* tension forces

(iii) Does perturb density/pressure

(iv) Propagates fastest perpendicular to ***B***

Linear MHD waves in non-uniform plasma

- Characteristic length scale defined by the inhomogeneity
- Equilibrium quantities are functions of position
- Continuum of resonant Alfvén and slow waves
- Discrete slow and fast modes; discrete Alfvén modes
- Efficient damping in non-ideal MHD
- MHD waves with mixed character and wave transformation

Linear MHD waves in non-uniform plasma

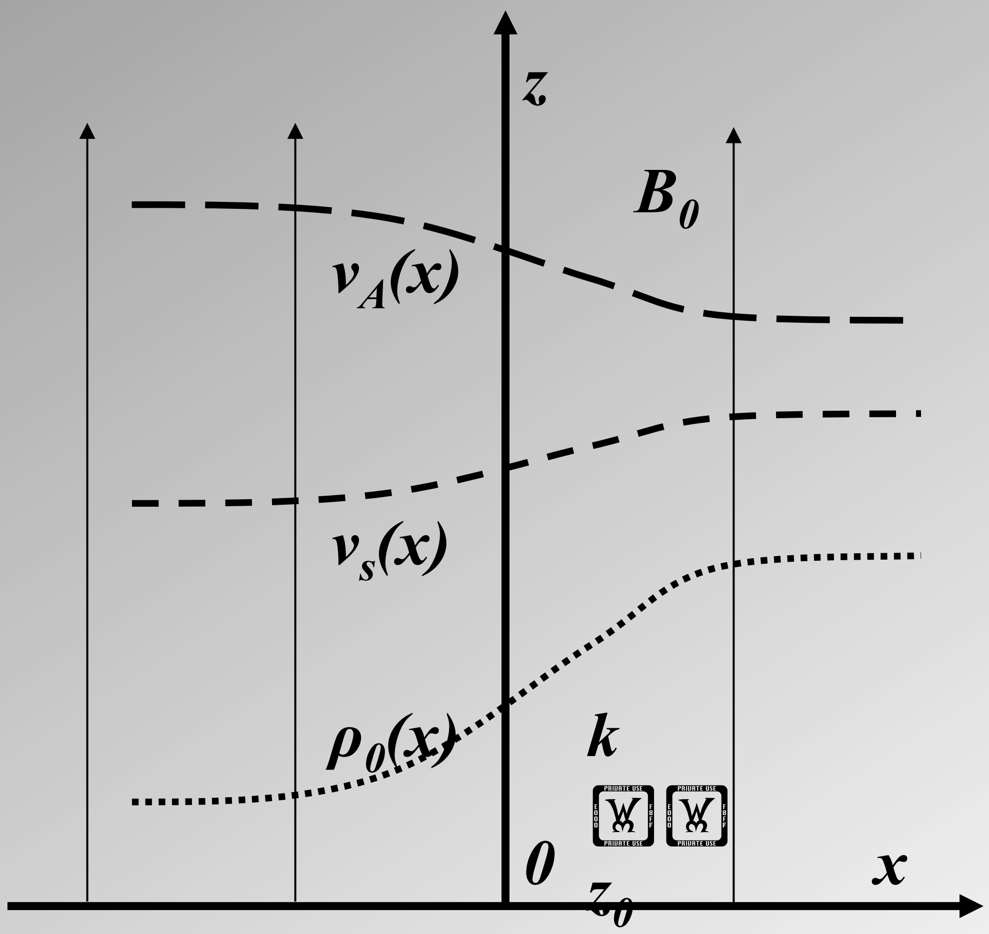
• Properties of MHD waves depend upon the angle between the wave vector and the magnetic field ➔ waves "feel" the direction of the field.

• When the magnetic field is not straight, Alfvén and slow waves should follow the field, because they are confined to the field.

• Even when the field is straight, inhomogeneities in the field absolute value, density and pressure affect the characteristic speeds of the waves (the Alfvén and the sound speeds) and, consequently, affect the waves.

• ➔ Guided propagation of MHD waves, linear coupling of different MHD modes, phase mixing of Alfvén waves, resonant absorption, appearance of wave dispersion, etc.

Linear MHD waves in non-uniform plasma



Linear MHD waves in non-uniform plasma

Total pressure balance: $$p_{\text{total}}(x) = p_0(x) + \frac{B_0^2(x)}{8\pi} = \text{const.}$$

Characteristic speeds:

- Alfvén speed $$v_{\text{A}}(x) = \frac{B_0(x)}{[4\pi\rho_0(x)]^{1/2}},$$
- Sound speed $$v_{\text{s}}(x) = [\gamma p_0(x)/\rho_0(x)]^{1/2},$$
- Tube (cusp) speed $$v_{\text{T}}(x) = \frac{v_s v_A}{[v_s^2 + v_A^2]^{1/2}} < v_s, v_A.$$

Linear MHD waves in non-uniform plasma

Fourier transform in homogeneous directions (y, z)

$$f(x)\exp[\mathrm{i}(\omega t - k_y y - k_z z)],$$

Boundary conditions at fixed x ➔ Dispersion Relation

$$D(\omega, k_y, k_z, [B_0(x), \rho_0(x) \text{ and } p_0(x)]) = 0.$$

Linear MHD waves in non-uniform plasma

Consider: $\delta/\delta y=0$, though V_y, $B_y \neq 0$ (i.e. 2.5D)

Alfvén modes

[perturbing V_y, B_y]

Magnetoacoustic modes

[perturbing V_x, V_z, B_x, B_z, ρ]

Linear MHD waves in non-uniform plasma

Magnetoacoustic modes are governed by:

$$\frac{d}{dx}\left(\frac{\Lambda(x)}{m_0^2(x)}\frac{dV_x}{dx}\right)-\Lambda(x)V_x=0,$$

$$\Lambda(x)=\rho_0(x)[\omega^2-k_z^2v_A^2(x)], \qquad m_0^2(x)=\frac{(k_z^2v_s^2-\omega^2)(k_z^2v_A^2-\omega^2)}{(v_A^2+v_s^2)(k_z^2v_T^2-\omega^2)},$$

+ B.C.s=eigenvalue problem. Eigenfunctions define transversal (x) structure of waves; eigenvalues define dispersion for waves.

Singularities: $\left\{ \begin{array}{ll} \text{Alfvén} & \omega/k_z=v_\mathrm{A}(x) \\ \text{Cusp} & \omega/k_z=v_\mathrm{T}(x) \end{array} \right.$ resonances!

Linear MHD waves in non-uniform plasma

Magnetoacoustic modes

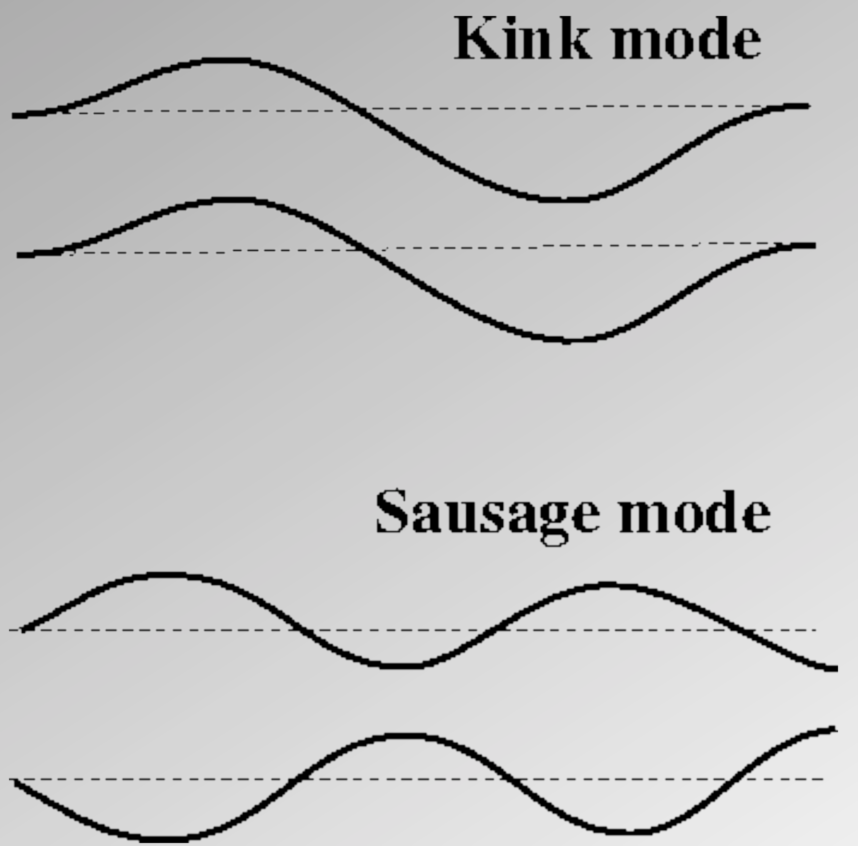
Evanescent solutions: *modes* or *trapped* or *guided* (or *ducted*) waves; Dispersion is determined by the ratio of the longitudinal wavelength to the characteristic spatial scale of inhomogeneity.

The modes can have different structures in *x* direction (inhomogeneity), which allows us to classify them:

- *kink* and *sausage* modes (perturbing or not perturbing the structure axis, respectively)
- *body* and *surface* modes (oscillating or evanescent inside the structure, respectively, and both evanescent outside the structure)

Linear MHD waves in non-uniform plasma

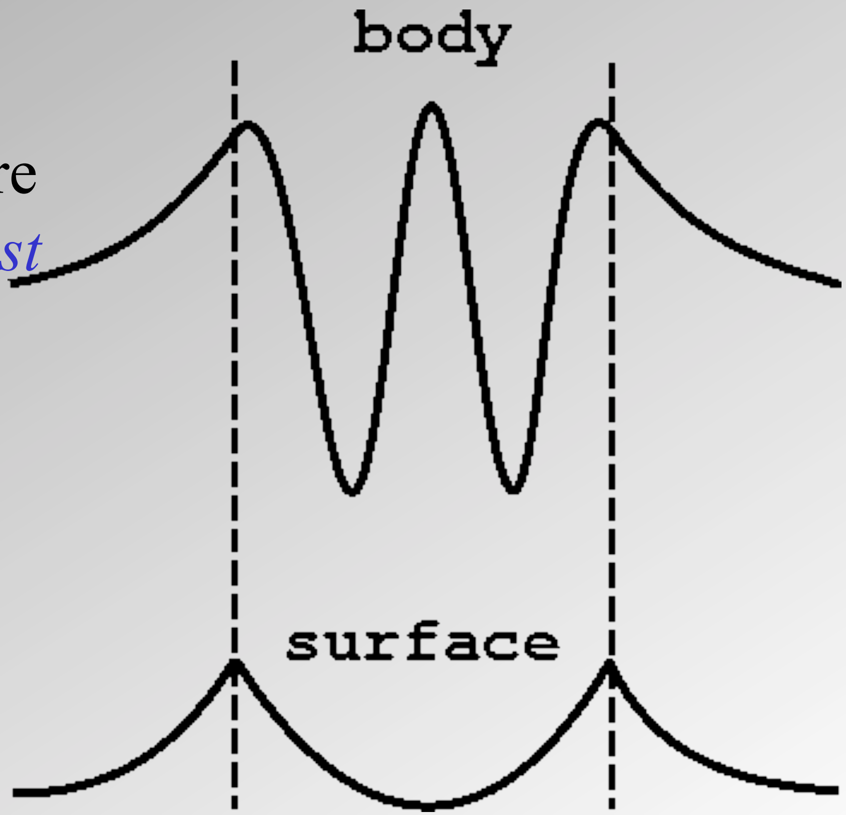
Magnetoacoustic modes



Linear MHD waves in non-uniform plasma

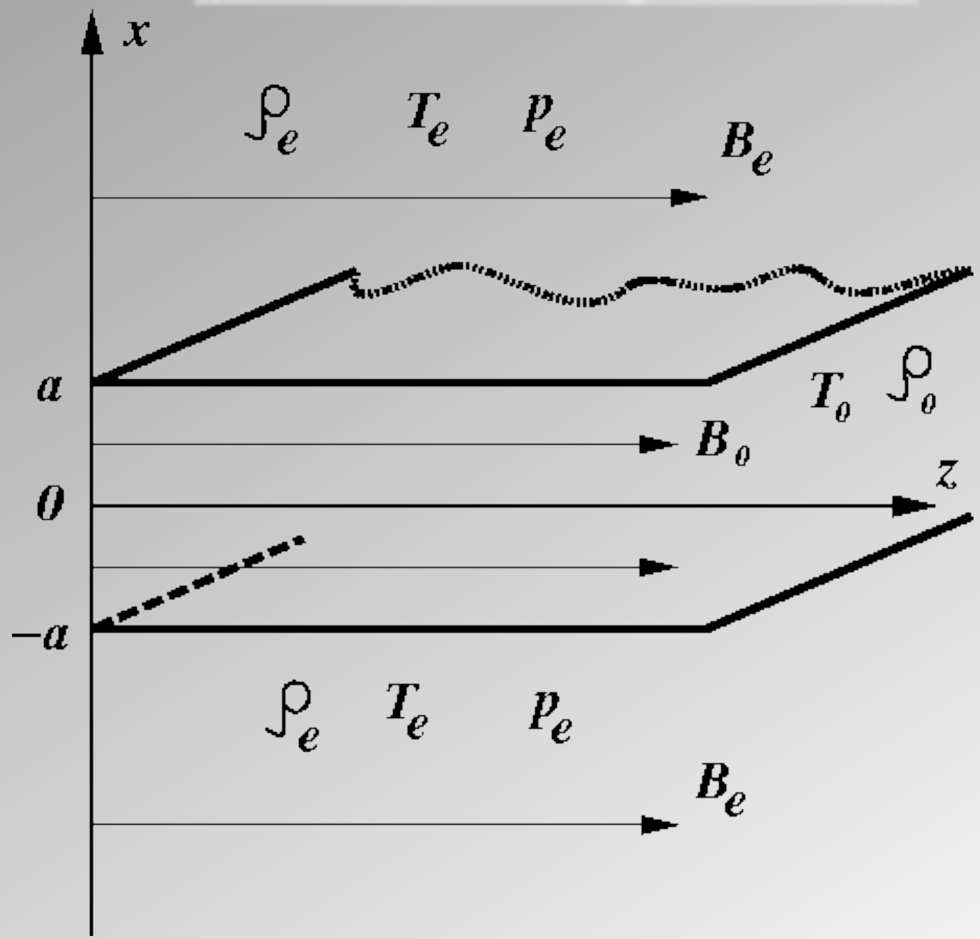
Magnetoacoustic modes

In addition, different modes of the same parity and transversal structure can be distinguished as *slow* and *fast* modes.



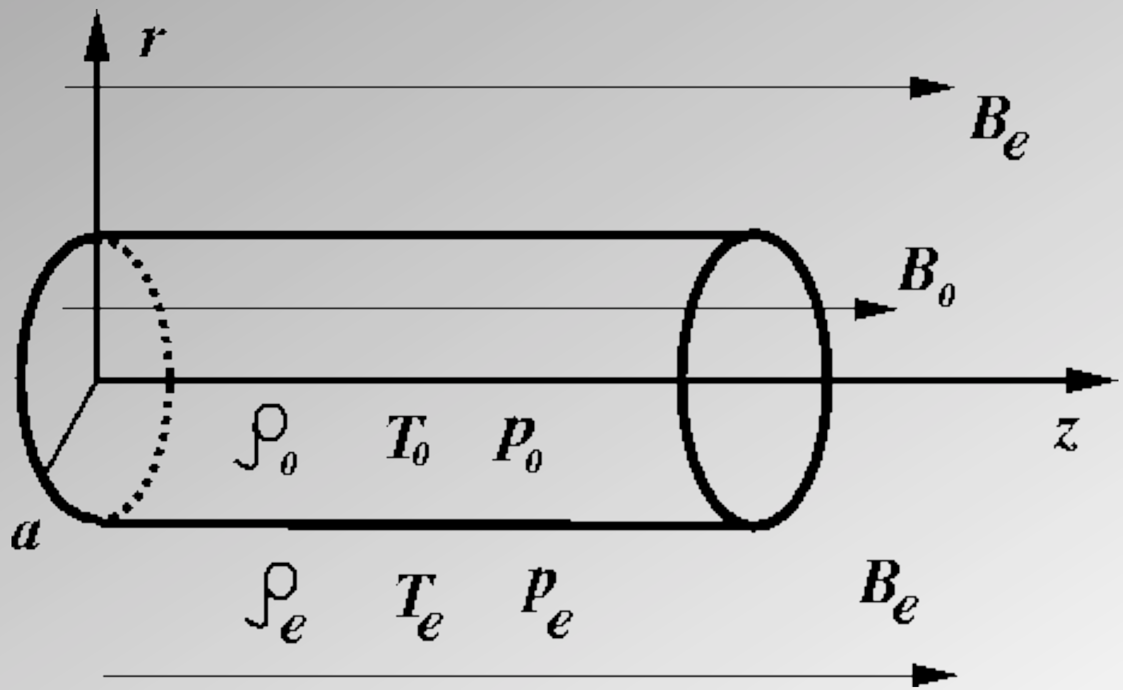
Different modes have different properties: dispersion relations, characteristic speeds, excitation conditions and observational manifestation.

MHD waves in magnetic tubes and slabs (structured plasma)



MHD waves in magnetic tubes and slabs (structured plasma)

- Linear motions of a compressible cylindrical/slab plasma



MHD waves in magnetic tubes and slabs

• Dispersion relation for slabs:

$$\rho_e(k_z^2 v_{Ae}^2 - \omega^2) m_0 \begin{Bmatrix} \tanh \\ \coth \end{Bmatrix} m_0 a + \rho_0 (k_z^2 v_{A0}^2 - \omega^2) m_e = 0$$

• where a is the slab semi-width and the tanh/coth terms correspond to the sausage/kink modes, respectively.

• DRs describe both surface ($m_0^2>0$) and body ($m_0^2<0$) waves.

• In ALL cases $m_e^2<0$ (non-leaky waves)

MHD waves in magnetic tubes and slabs

- Dispersion relation for tubes

$$\rho_0(k^2 v_A^2 - \omega^2) m_e \frac{K_n'(m_e a)}{K_n(m_e a)} = \rho_e(k^2 v_{Ae}^2 - \omega^2) m_0 \frac{I_n'(m_o a)}{I_n(m_o a)}$$

$m_0^2 > 0$ → surface waves

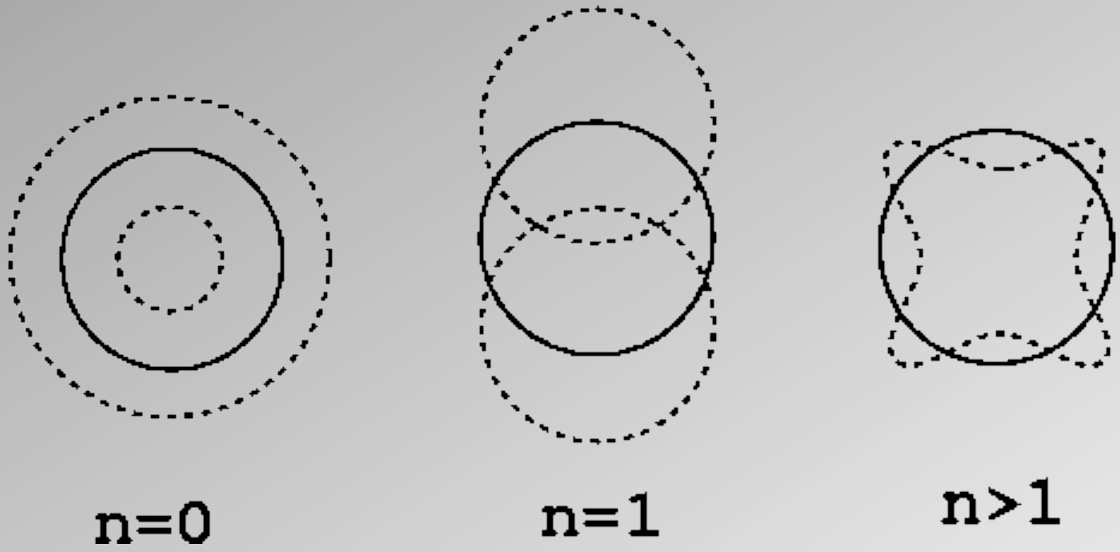
$$\rho_0(k^2 v_A^2 - \omega^2) m_e \frac{K_n'(m_e a)}{K_n(m_e a)} = \rho_e(k^2 v_{Ae}^2 - \omega^2) n_0 \frac{J_n'(n_o a)}{J_n(n_o a)}$$

$m_0^2 = -n_0^2 < 0$ → body waves. Note n=0 refers to sausage, n=1 to kink modes, etc.

Roberts (1981), Edwin & Roberts (1983)

MHD waves in magnetic tubes and slabs

• The number *n* determines the mode structure:



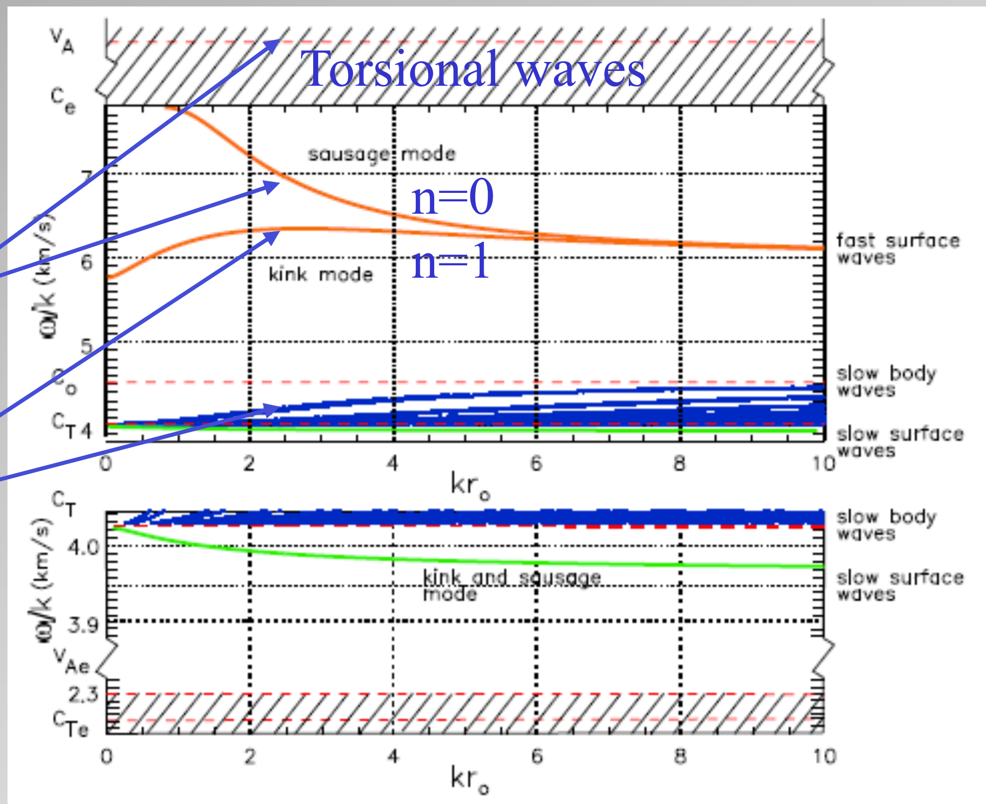
• Therefore, the modes with *n*=0 are sausage modes and with *n*=1 are kink, n>1: "flute" modes.

Theory of tube oscillations

Solutions to DR for lower atmosphere magnetic tubes

Main modes:

- (Alfvenic) torsional (incompressible)
- fast sausage (|B|, ρ)
- fast kink (almost incompressible)
- slow (acoustic) type (ρ, v)



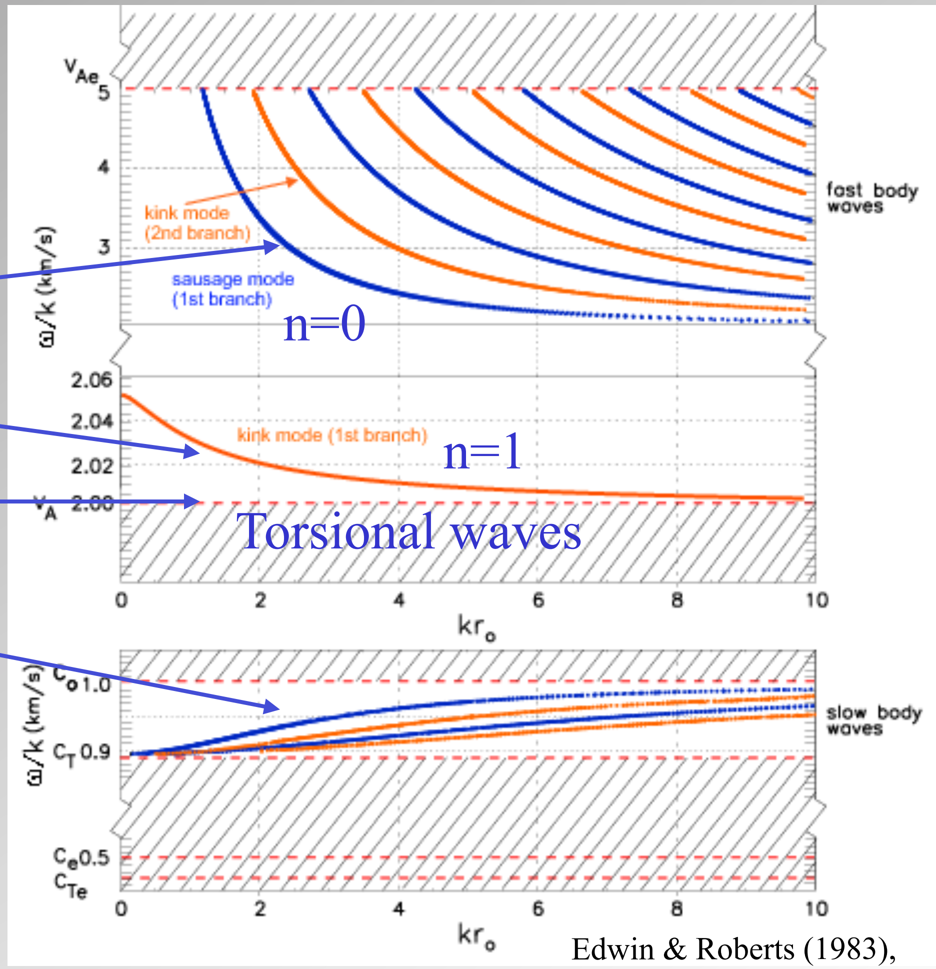
Edwin & Roberts (1983), Erdélyi (2008)

Theory of tube oscillations

Solutions to DR for coronal loops

Main modes:

- fast sausage (|B|, ρ)
- fast kink (almost incompressible)
- (Alfvenic) torsional (incompressible)
- slow (acoustic) type (ρ, v)



Edwin & Roberts (1983), Erdélyi (2008)

Theory of tube oscillations

Solutions to l ... **cillation periods**

... **onal loops**

Main modes:

- sausage ... s: P = 0.1 - 5 s
- kink ... P = 1.4 - 14 min
- (Alfvenic) t ... es: P = 1 s
- slow (acoust ... P = 7 -70 min

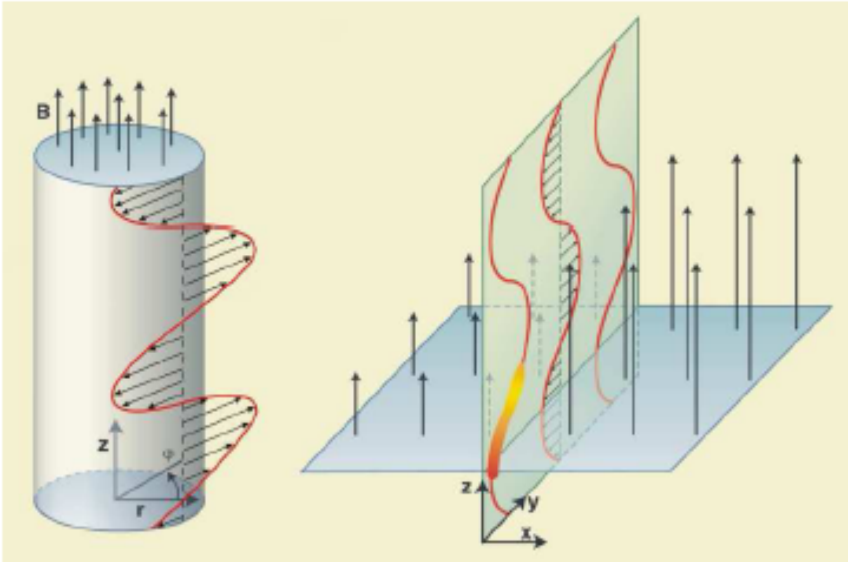


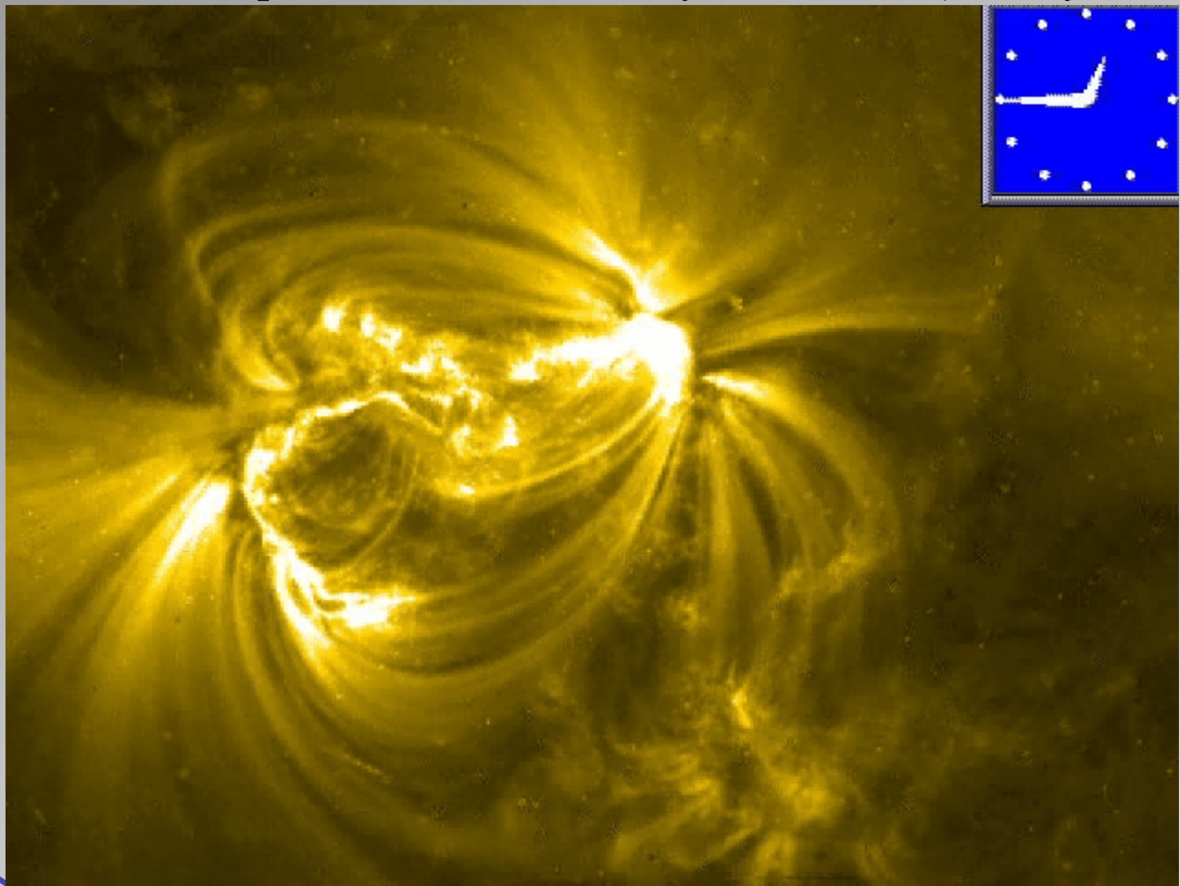
Figure 5.4: *Left:* Magnetic flux tube showing a snapshot of Alfvén wave perturbation propagating in the longitudinal z-direction along field lines at the tube boundary. At a given height the Alfénic perturbations are torsional oscillations, i.e. oscillations are in the φ-direction, perpendicular to the background field. *Right:* Snapshot showing Alfvén waves propagating along a magnetic discontinuity. Again, the key feature to note is that Alfvénic perturbations are *within* the magnetic surface (yz-plane) at the discontinuity, perpendicular to the background field (y-direction), while the waves themselves propagate along the field lines (z-direction).

… KG

Aschwanden (2003), Wang (2004)

Standing kink (transversal) modes #1

- TRACE: Loop oscillation excited by M4.6 flare (14 July 1998)

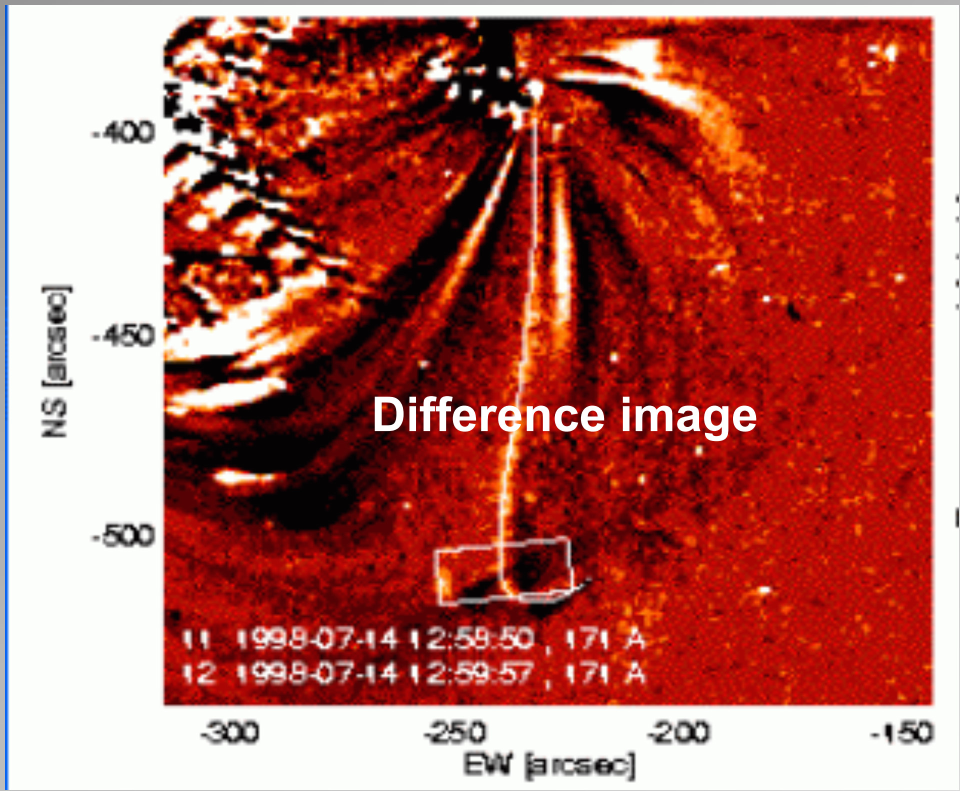


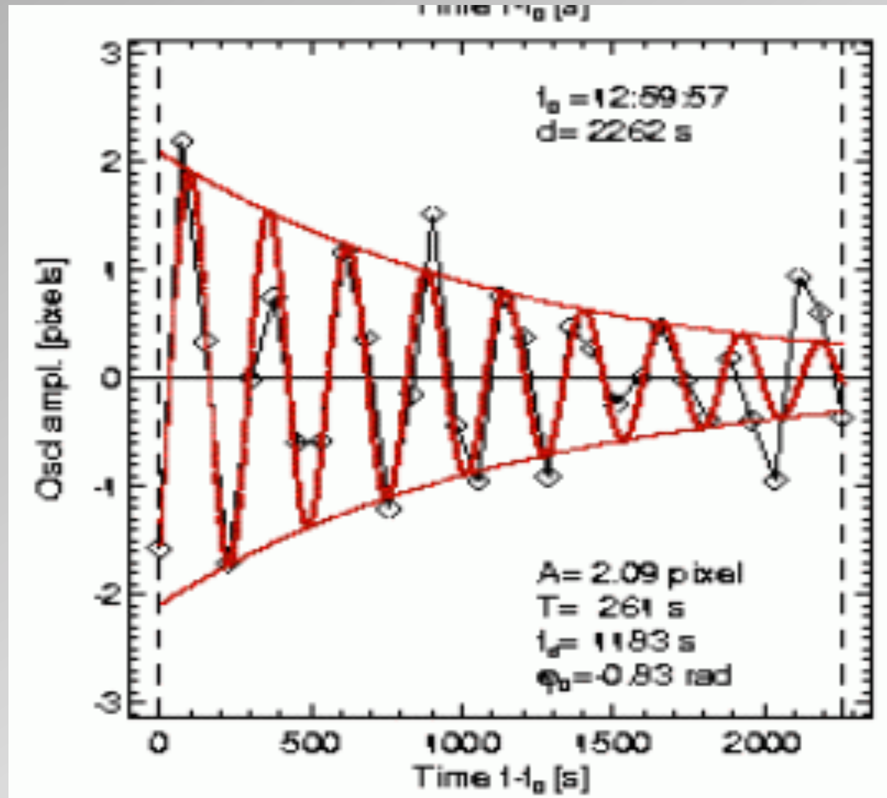
Movie in TRACE 171 A

Occurrence rate**:** 17/255 flares with transverse oscillation

Schrijver et al. 2002

Standing kink (transversal) modes #1

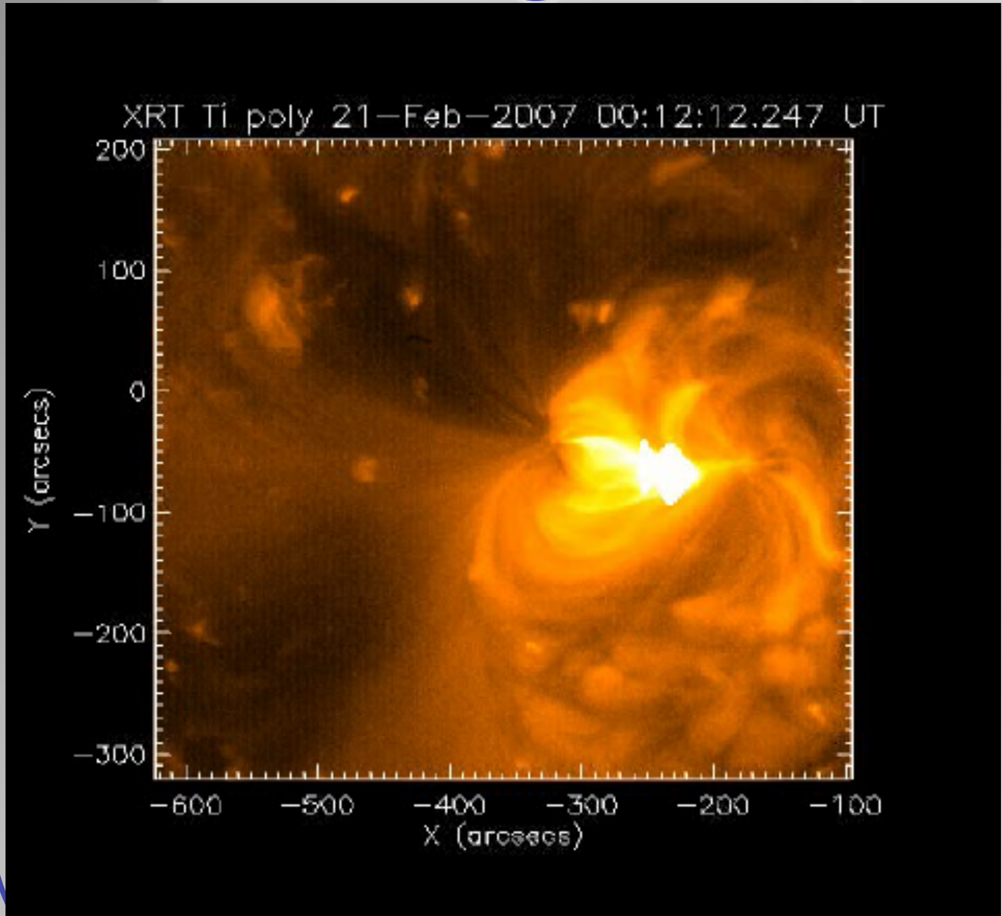




Damped oscillations are fitted by $A(t) = A_0 \sin(\omega t+\varphi)\, e^{-\lambda t}$ ⇒ Amplitude, Period, Decay time

Aschwanden et al. 2002

Standing kink (transversal) modes #2



Movie in Hinode XRT Ti poly

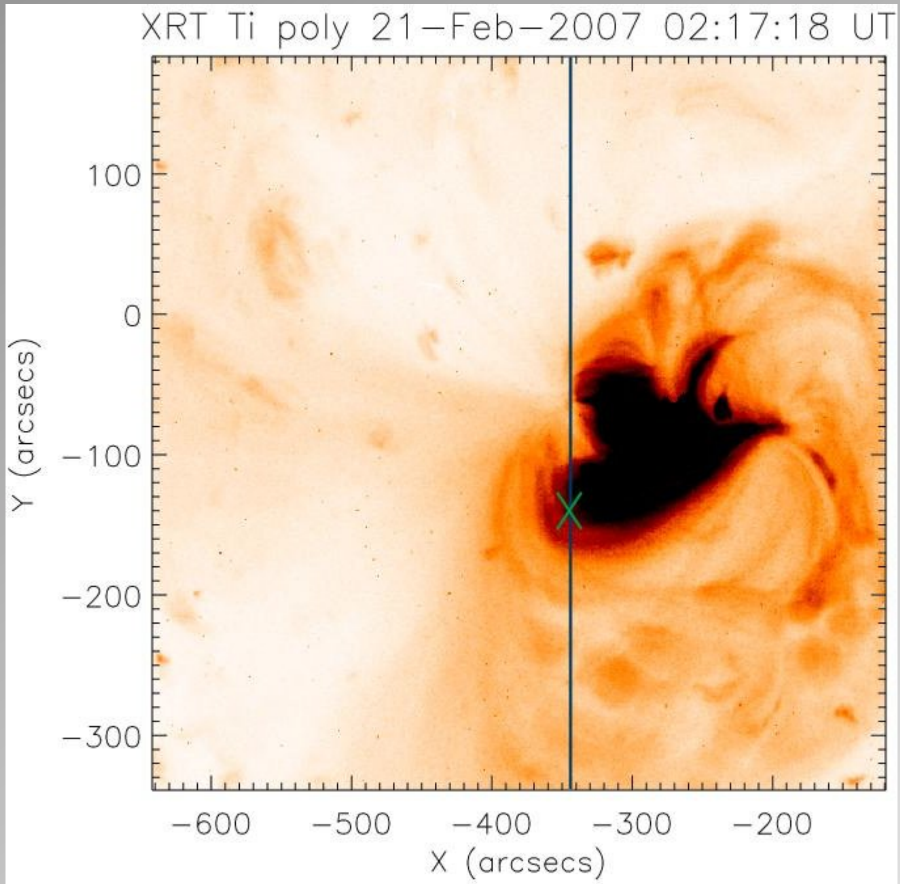
- There is nothing to prevent oscillations in EUV loops
- Oscillations are best seen in Doppler shift

➔ let's analyse Hinode data!

- Loop oscillation excitation unclear (21 February 2007)

Erdélyi & Taroyan 2008

Standing kink (transversal) modes #2

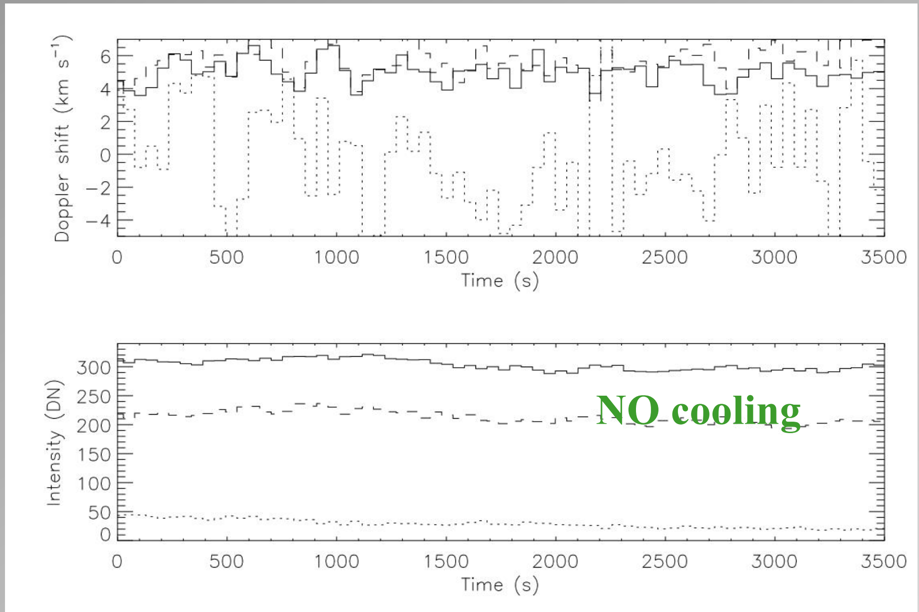


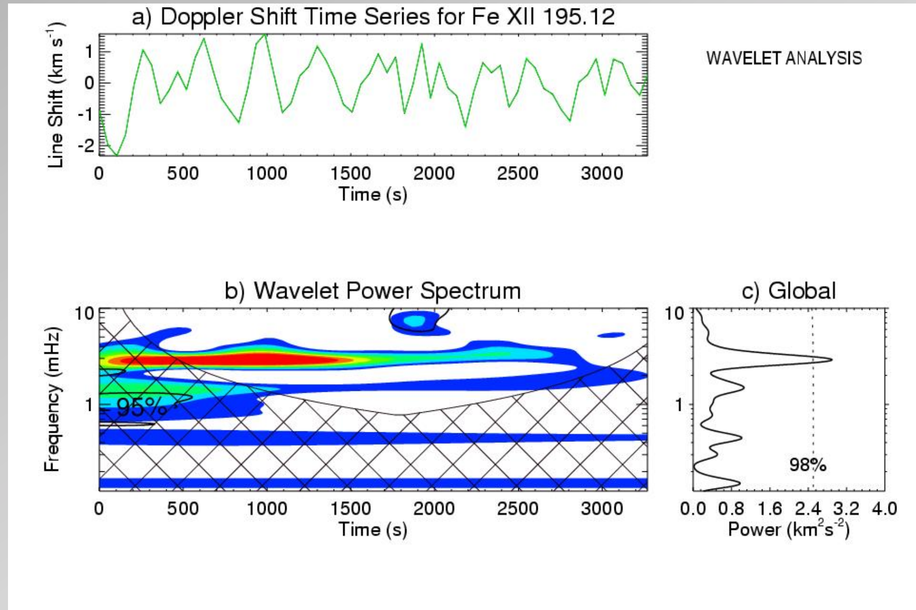
- **Slit close to apex ➔ detected wave motion is likely to be transversal**
- **No EIS images were available**

- Loop oscillation excitation unclear (21 February 2007)

Erdélyi & Taroyan 2008

Standing kink (transversal) modes #2





Doppler shift ➔ interpreted as example of kink waves in EUV loop

Doppler shift oscillations f=3 mHz

Magnetic field: B=~10 G

- Loop oscillation excitation unclear (21 February 2007)

Erdélyi & Taroyan 2008

Standing kink (transversal) modes

Application of MHD coronal seismology

Measurements of coronal magnetic field strength

For a standing global (fundamental) kink mode

$$B = 18\left(\frac{L}{100\ \mathrm{Mm}}\right)\left(\frac{P}{300\ \mathrm{s}}\right)^{-1}\sqrt{\frac{n_{\mathrm{loop}}}{10^9\ \mathrm{cm}^{-3}}}\ [\mathrm{G}]$$

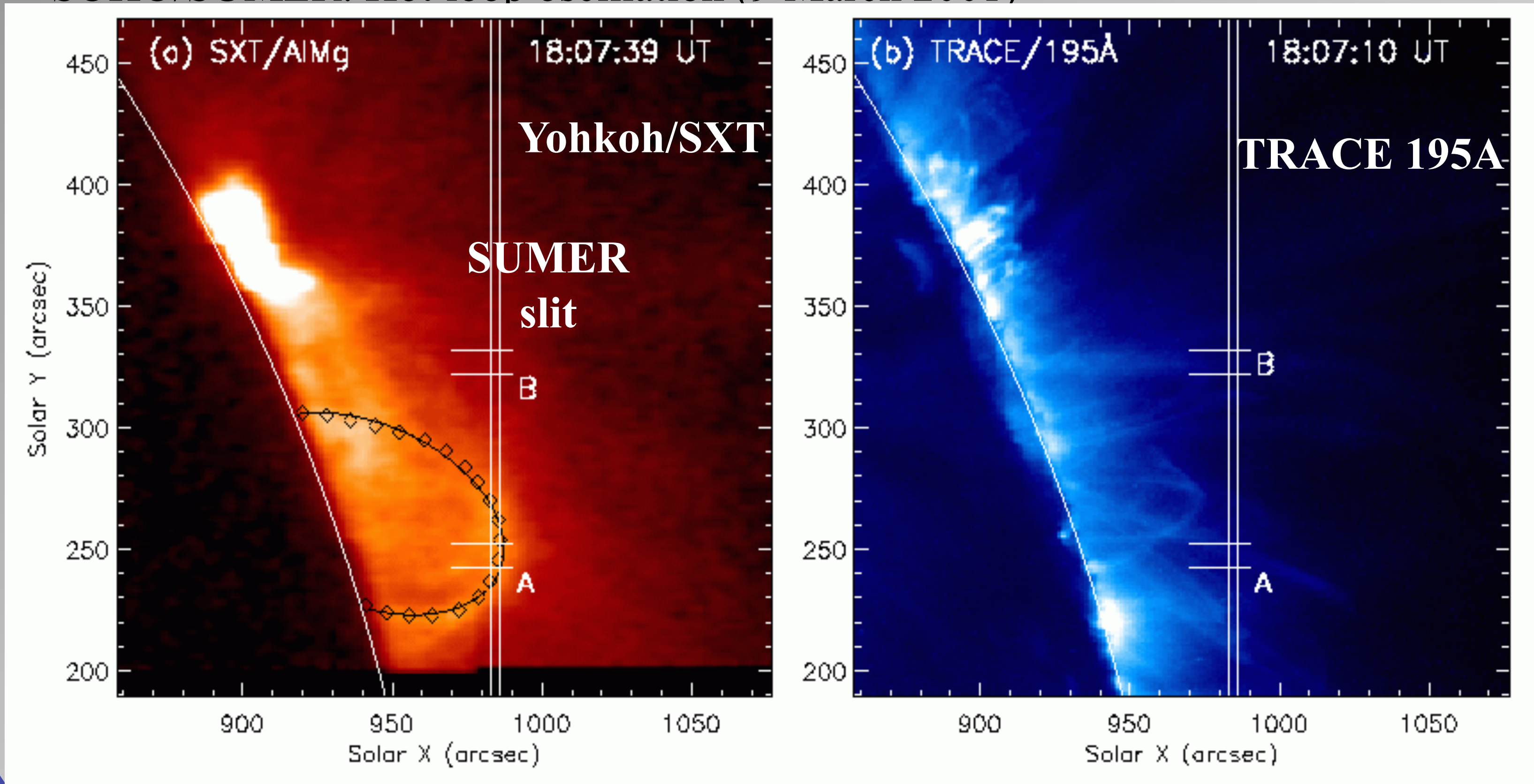
For loop length: 2L=200 Mm,
osci. period: P = 5 min,
loop density $n_{loop}=10^9$ cm,

Obtain: mean: B = 18 G
range: B = 3 - 30 G

Nakariakov & Ofman (2001): B = 4 - 30 G for 2 cases
Aschwanden et al . (2002): B = 3 - 30 G for 26 cases

Standing slow modes #1

• SOHO/SUMER: Hot loop oscillation (9 March 2001)

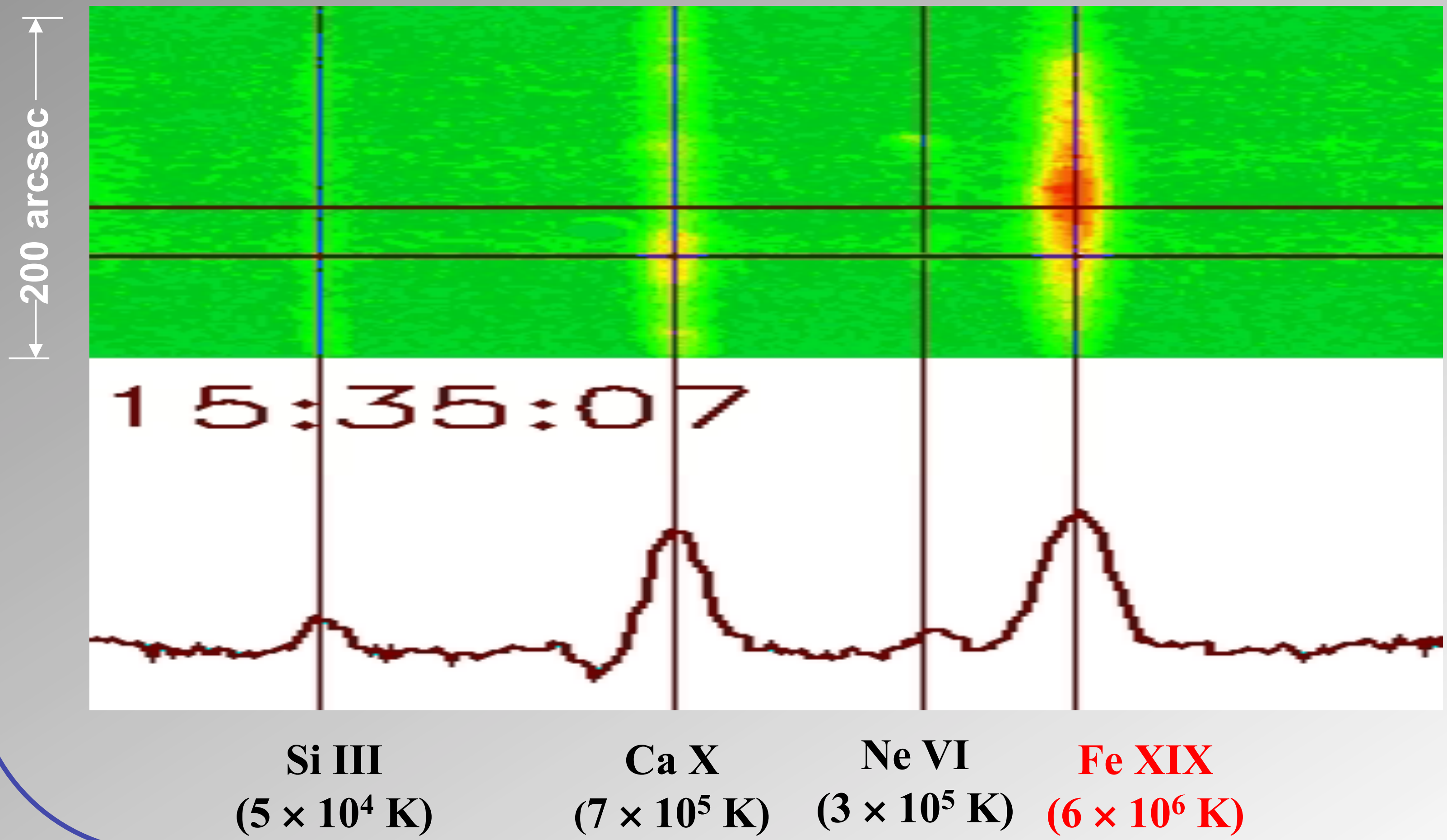


Flare was not found

Wang et al. 2002

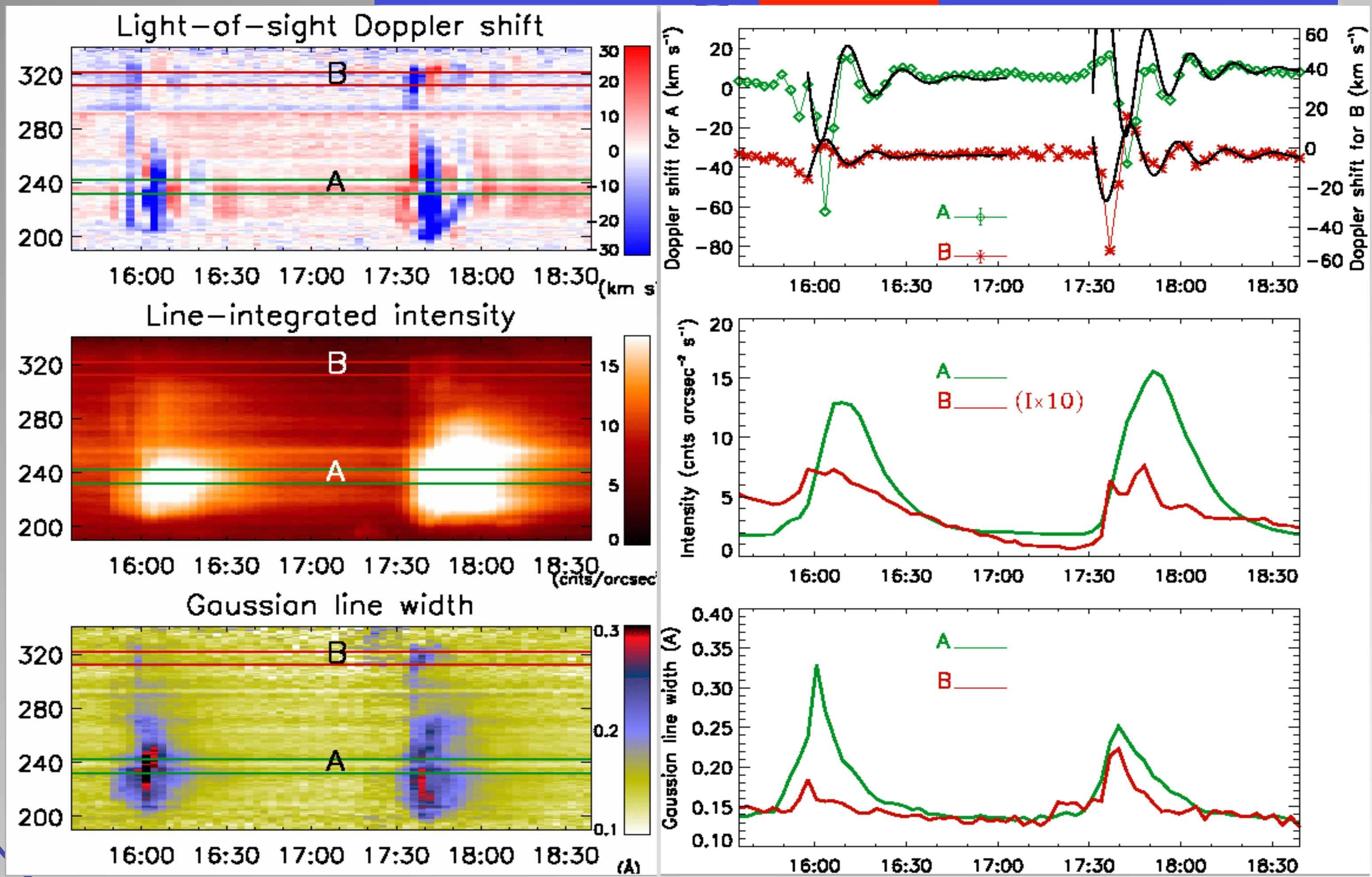
Standing slow modes #1

- Window 1100-1140 Å, lines of T=10^4-10^7 K



Wang et al. 2002

Standing slow modes #1



P=14 – 18 min √

T_d=12 – 19 min

Phase speed $V_p = 2L/P = 240 – 380$ km/s √

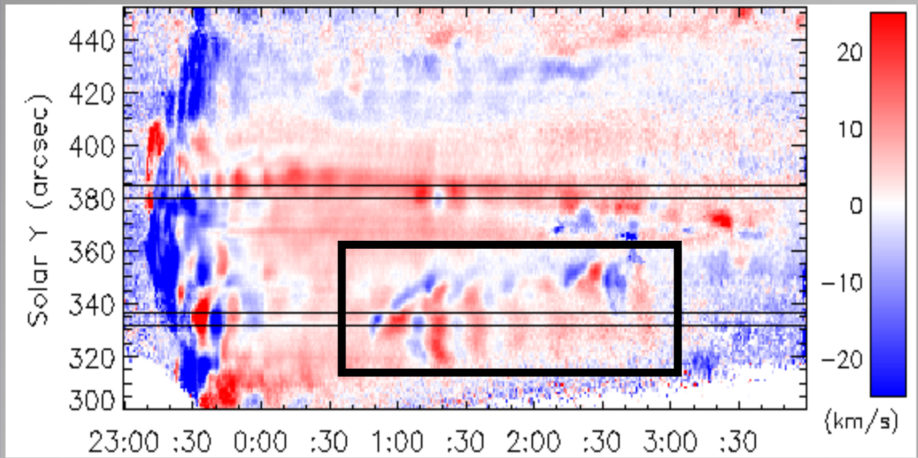
Sound speed: $v_s = 380$ km/s at T = 6.3 MK

Wang et al. 2002

Standing slow modes #2

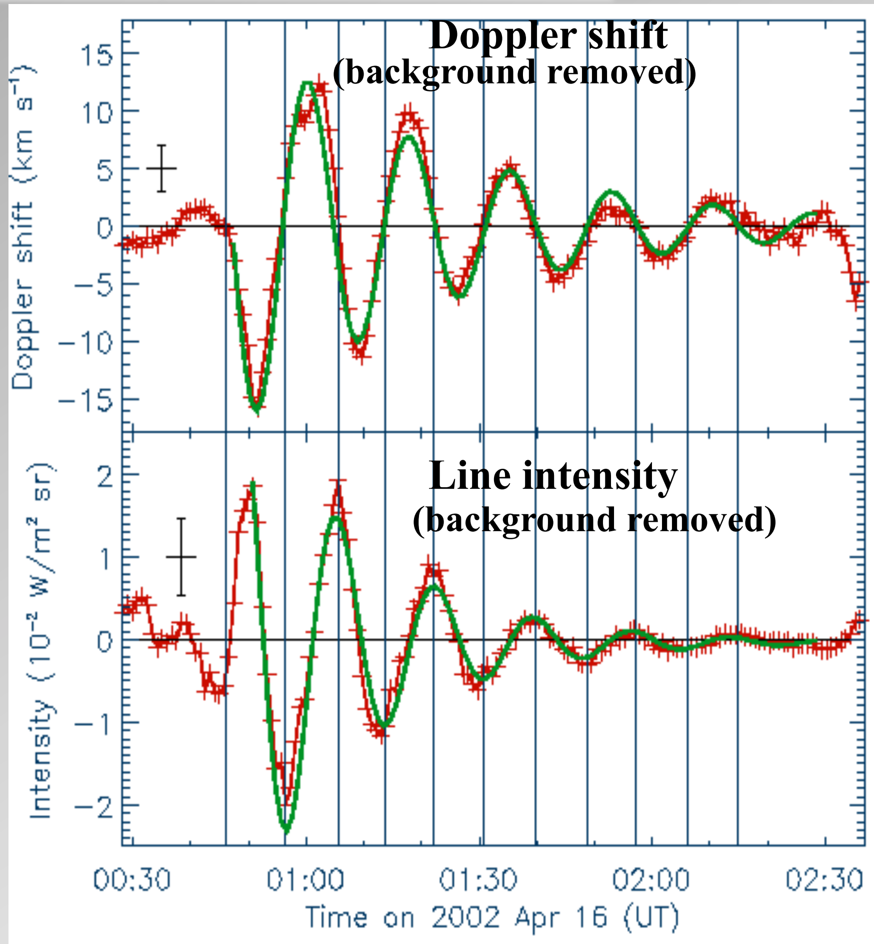
• Case of 16 April 2002

Doppler shift time series in Fe XIX



Doppler shift oscillations   P=17.6 min √
Line intensity oscillations P=17.1 min √

1/4 phase difference between velocity and intensity √

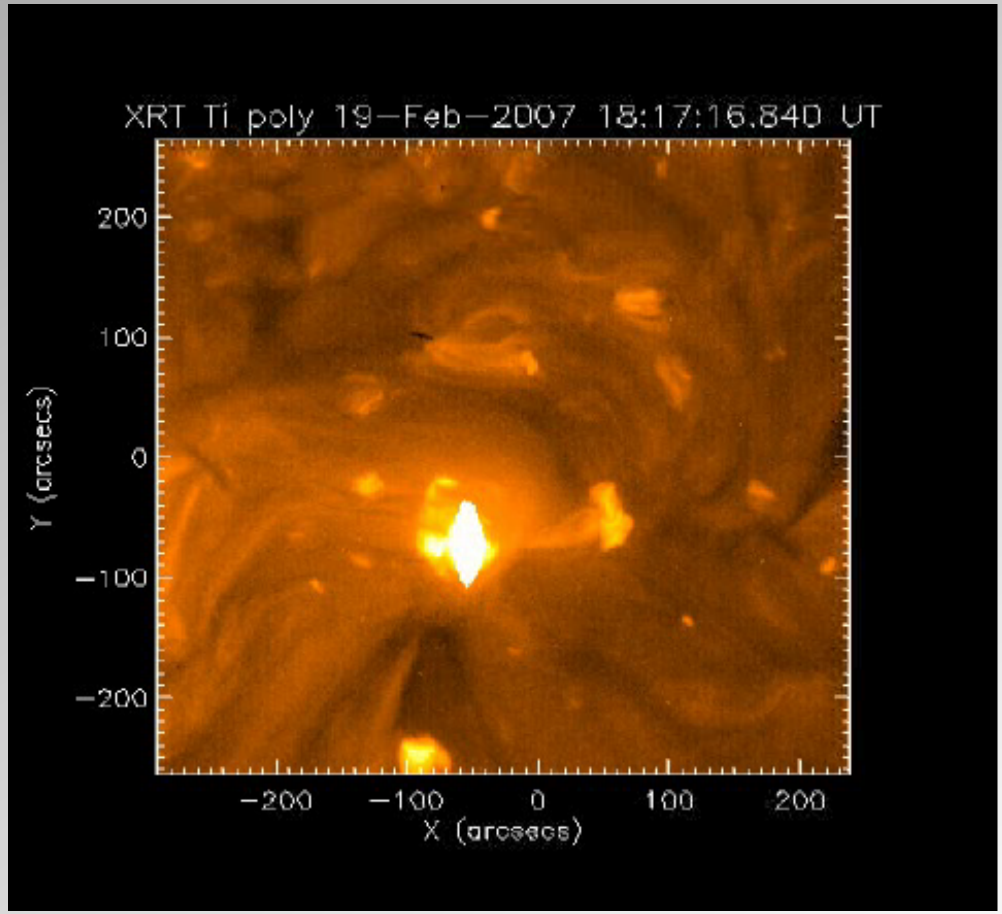


Wang et al. 2003a

Standing slow modes #3

- There is nothing to prevent oscillations in EUV loops
- Oscillations are best seen in Doppler shift

➔ let's analyse Hinode data!

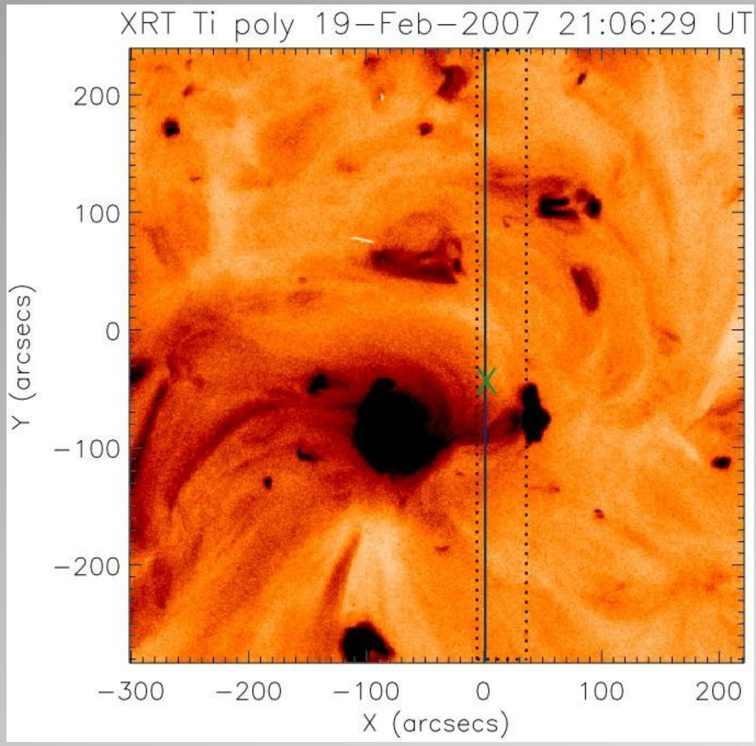


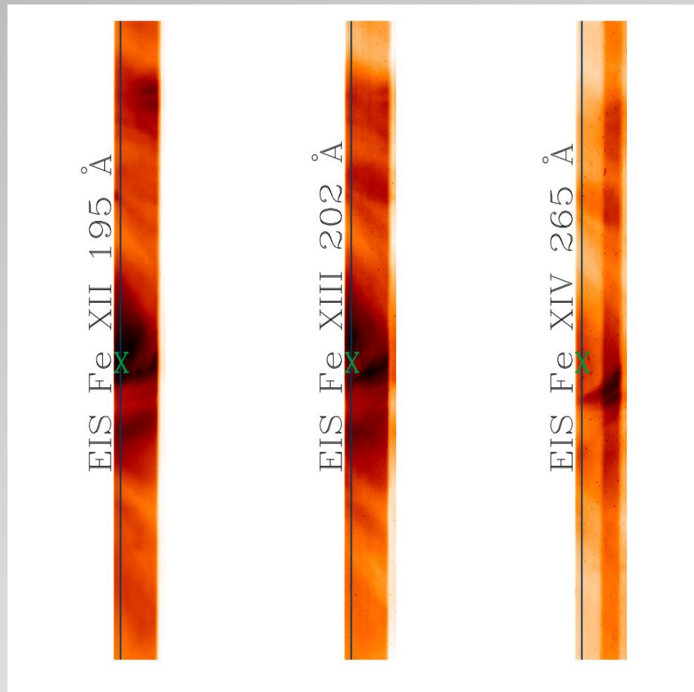
Erdélyi & Taroyan 2008

Standing slow modes #3

- Case of 19 February 2007

XRT & Hinode/EIS observations



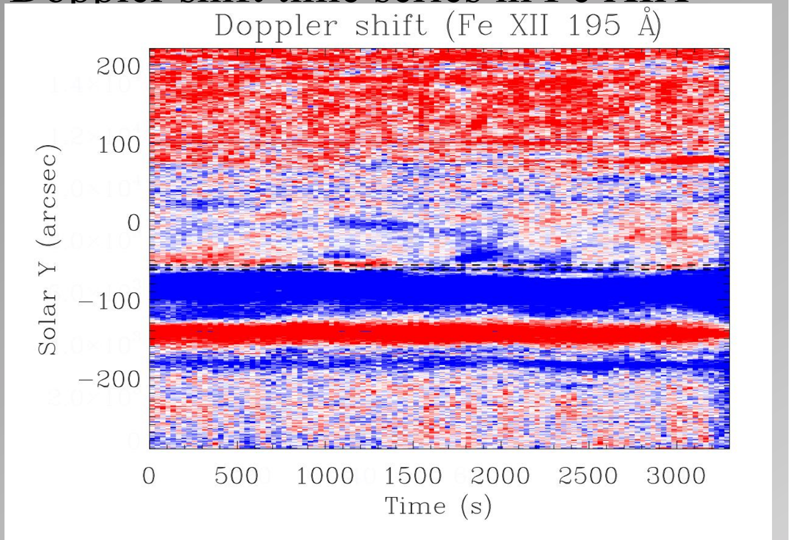


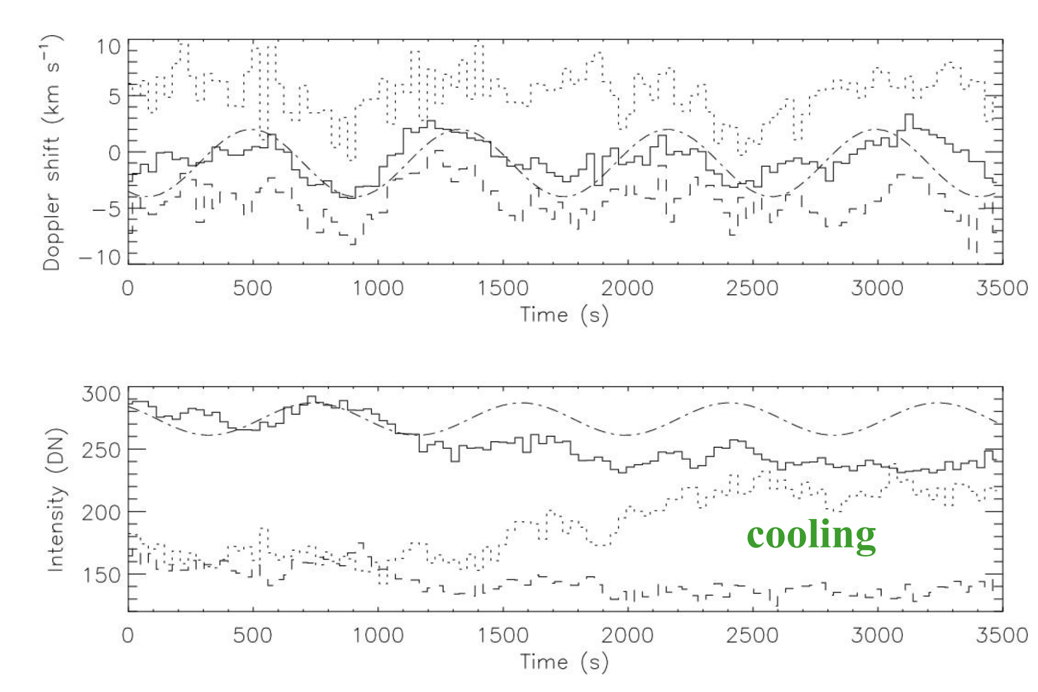
Erdélyi & Taroyan 2008

Standing slow modes #3

- Case of 19 February 2007

Doppler shift time series in Fe XIX





Doppler shift oscillations f=1.2 mHz √
Line intensity oscillations f=1.2 mHz √

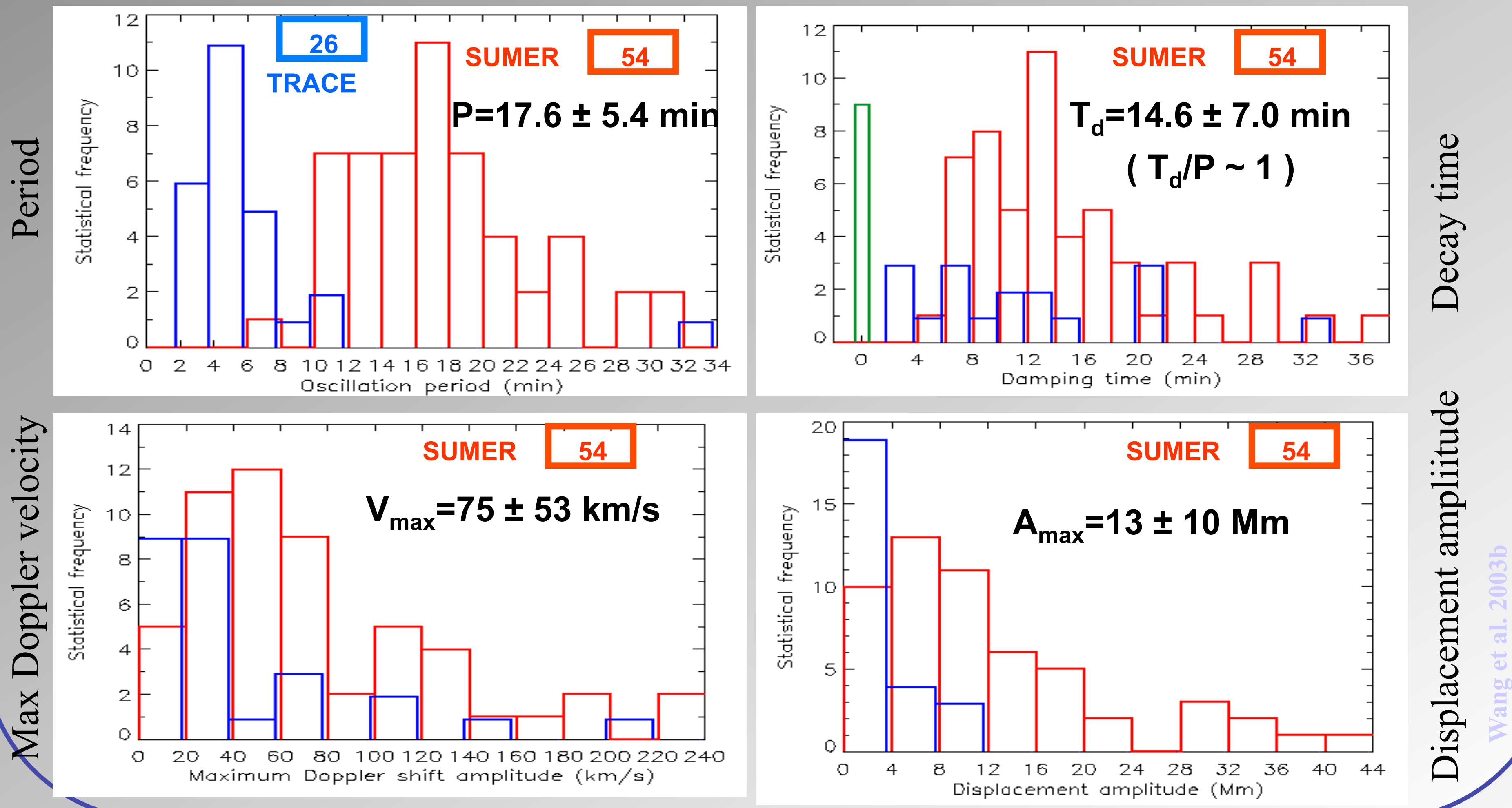
1/4 phase difference between velocity and intensity √

Quarter period phase shift! ➔ 1st example of acoustic waves in EUV loop

Erdélyi & Taroyan 2008

Solar Physics & Space Plasma Research Center (SP^2RC)

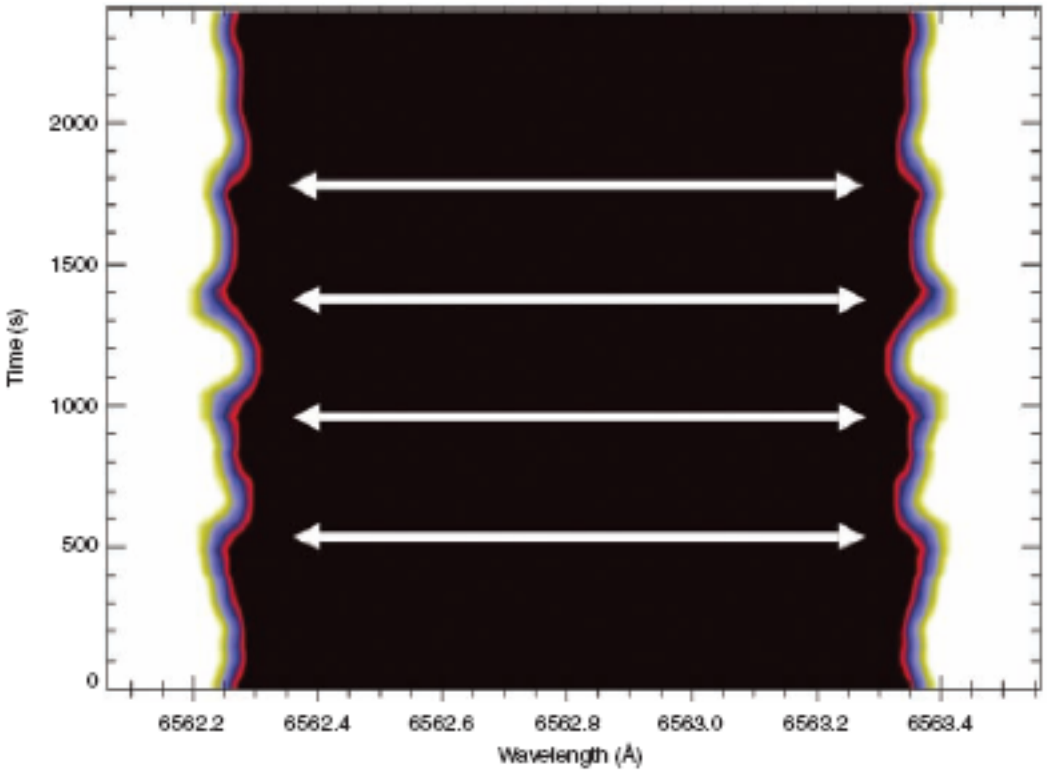
Overview of measurements



Alfvén modes

- SST: Chromospheric bright point oscillations

Fig. 2. A wavelength-versus-time plot of the Hα profile showing the variation of line width at FWHM as a function of time. The arrows indicate the positions of maximum amplitude of a 420-s periodicity associated with the bright-point group located at (−10 arc sec, 10 arc sec) in Fig. 1. The torsional motion of the Alfvénic perturbations creates nonthermal broadening that is visible in the Hα line profile. The peak-to-peak velocity is ≈3.0 km s^{-1} (≈65 mÅ). For an inclination angle of 35°, the absolute velocity amplitude is ≈2.6 km s^{-1}.



Intensity oscillations were not found!

Jess et al. 2009

Alfvén modes

• SST: Chromospheric bright point oscillations

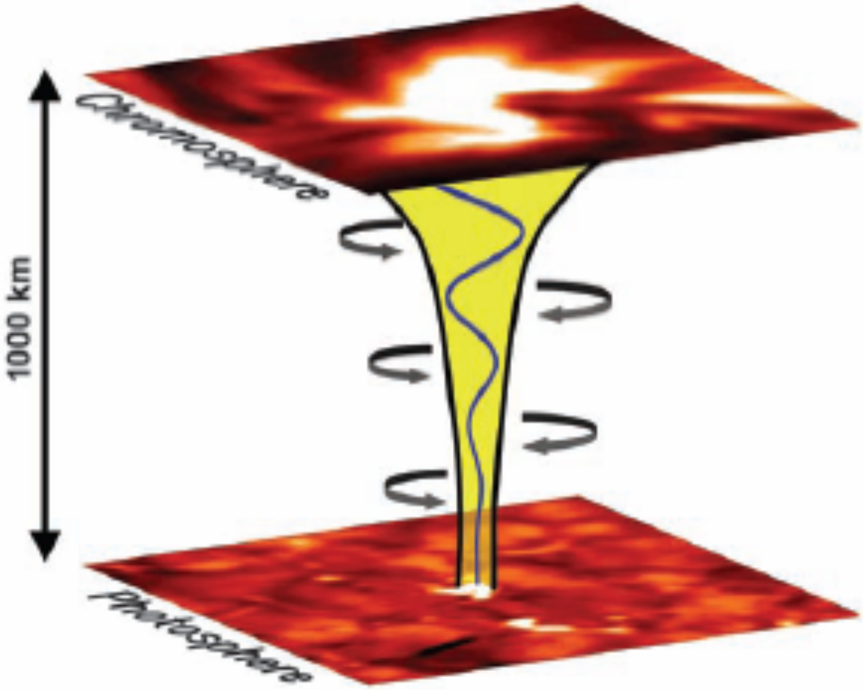


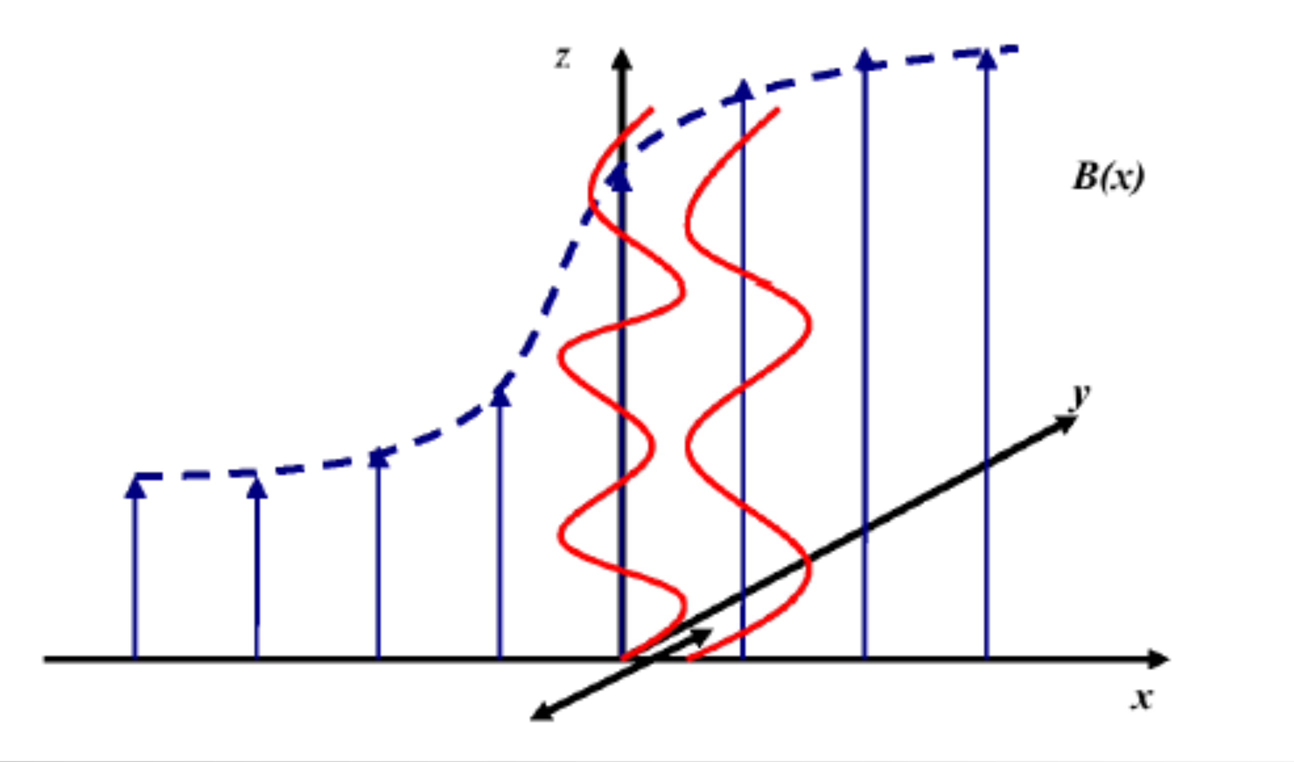
Fig. 3. Expanding magnetic flux tube sandwiched between photospheric and chromospheric intensity images obtained with the SST, undergoing a torsional Alfvénic perturbation and generating a wave that propagates longitudinally in the vertical direction. At a given position along the flux tube, the Alfvénic displacements are torsional oscillations that remain perpendicular to the direction of propagation and magnetic field outlining constant magnetic surfaces. The largest FWHM will be produced when the torsional velocity is at its maximum (at zero displacement from the equilibrium position). The figure is not to scale.

Intensity oscillations were not found!

Jess et al. 2009

Alfvén wave phase mixing

• 1D field-aligned inhomogeneity, Alfvén waves …



Alfvén wave phase mixing

• In 1D field-aligned inhomogeneity, Alfvén waves are described by

$$\left(\frac{\partial^2}{\partial t^2} - v_A^2(x)\frac{\partial^2}{\partial z^2}\right)V_y = 0,$$

• Solution: $V_y = \Psi(x) f(z \mp v_A(x)t),$

where f(x) and Ψ(x) are functions prescribed by the initial profile of the wave.

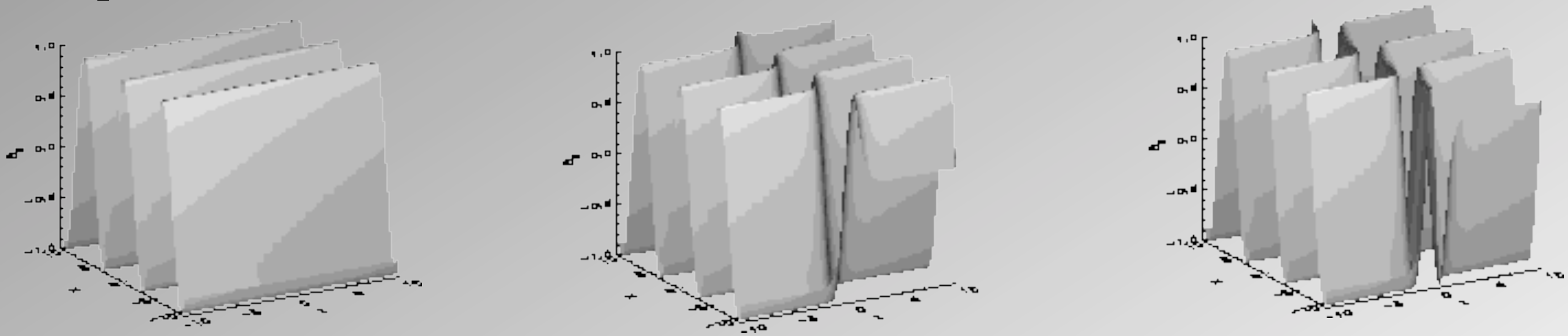
• Solution shows the Alfvén waves propagate on diff. magnetic surfaces (corresp. to different values of x) with different speeds (equal to the local Alfvén speed $v_A(x)$.

• If the wave is initially plane in the x direction, it gets gradually inclined.

Alfvén wave phase mixing

• Consider the evolution of an initially (t=0) plane Alfvén wave on the smooth magnetic interface with the profile

• Snapshots at t=0, 2 and 4: $v_A(x) = 1 + \tanh x$



• In some time, perturbations of different magnetic surfaces become uncorrelated with each other. Because the Alfvén wave is not able to propagate across the magnetic field (in the x direction), k_z=const and $k_x \rightarrow \infty$. This is phase mixing (Heyvaerts & Priest 1983)!

• In the presence of small but finite viscosity or resistivity: efficient damping.

Resonant absorption

- Inhomogeneous plasmas: natural behaviour
- Easy wave energy transfer resulting in heating
- Condition to occur: $\omega_{driver} = \omega_{local}$
- Versatile as could/may/viable to explain:

  - local/atmospheric heating
  - power loss of acoustic waves in sunspots
  - damping of helioseismic (*p*/*f*/*g*) eigenmodes
  - energisation of MHD waves in magneto/heliosphere

The Concept of Resonant Absorption

¤ **Global modes resonantly interact with local MHD modes**

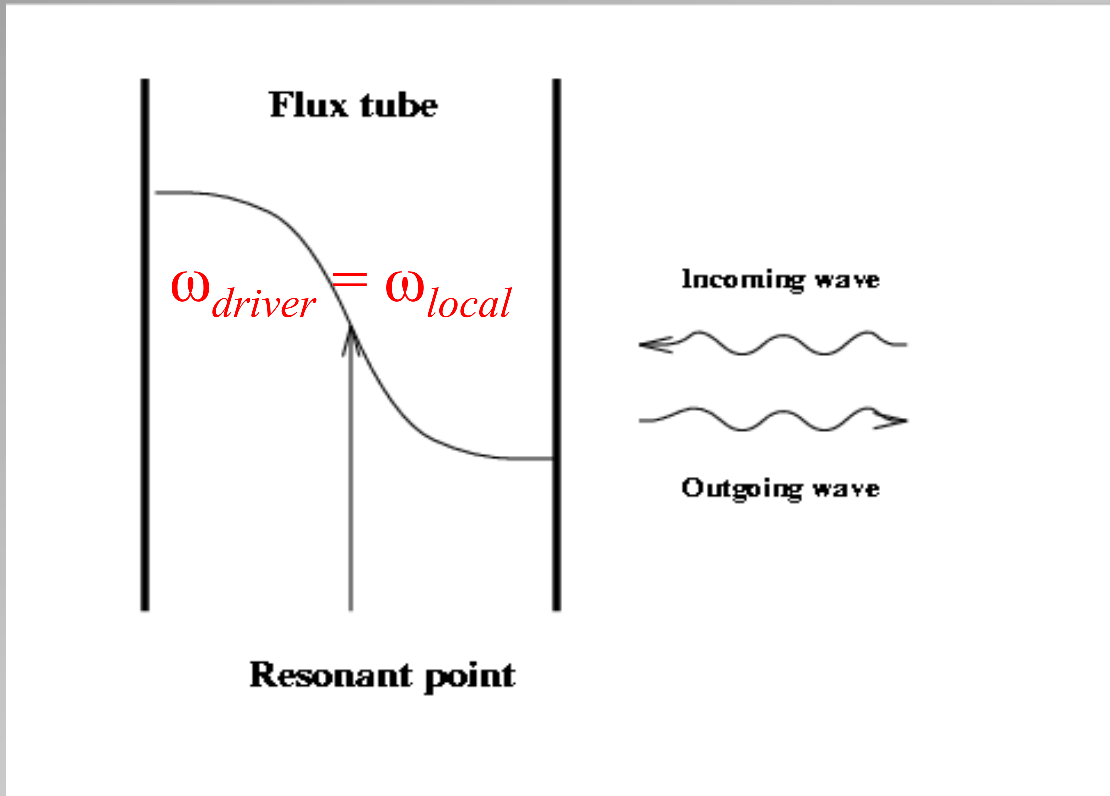
↓

¤ **Dissipation** ← **Steady state**

↓

¤ **Damping of global oscillations**

The Concept of RA



Ideal MHD equations singular ➪ dissipation ➪ heating

Connection Formulae

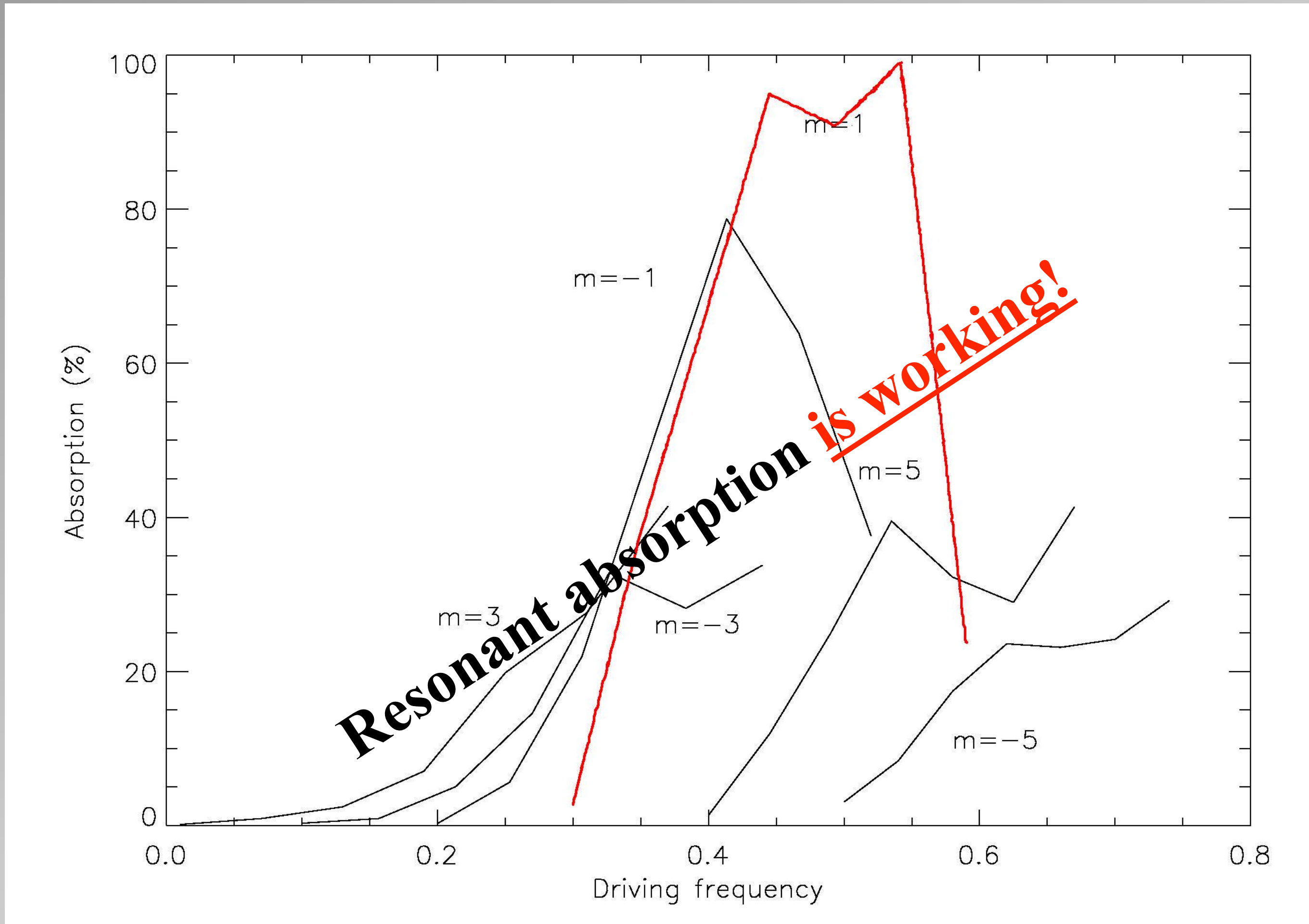
Where does RA stand?

- **MHD** (e.g., Alfvén) **wave dissipation**
- Resistive dissipation of D.C. electric currents
- Selective decay of turbulent cascade of magnetic fields
- (corona: non-Maxwellian particle distribution generated in the chromosphere – transition zone)

- Resonant absorption: Ionson '78, Rae & Roberts '82, Davila '87, Hollweg '84, Poedts *et al*. 1989; Ruderman *et al*. 1997ab; Erdélyi & Goossens '94, '95, '96; Ofman *et al*. '94, 95abc, 98; Ballai *et al*. '98ab, '00ab, '02, Balthazor & Erdélyi '00, Erdélyi *et al*. '01, '02, etc., Ruderman & Roberts 2002

•New: reconnection driven resonant waves (Roussev et al. '01)

Application to coronal heating



Concept of connection formulae

- Hollweg '87, Sakurai *et al*. '91, Goossens *et al*. '92, '95, Erdélyi *et al*. '95, Erdélyi '97

$$D\frac{d(r\xi_r)}{dr} = C_1 r\xi_r - C_2 r P_1$$

$$D\frac{dP_1}{dr} = C_3\xi_r - C_1 P_1$$

$$D = \rho(c^2 + v_A^2)(\Omega^2 - \omega_A^2)(\Omega^2 - \omega_C^2)$$

Mobile regular singularities: $\Omega^2(r)=\omega^2_{A/C}(r)$

Concept of connection formulae

- Driven problem ➔ ω is prescribed
- Eigenvalue problem ➔ ω is searched for

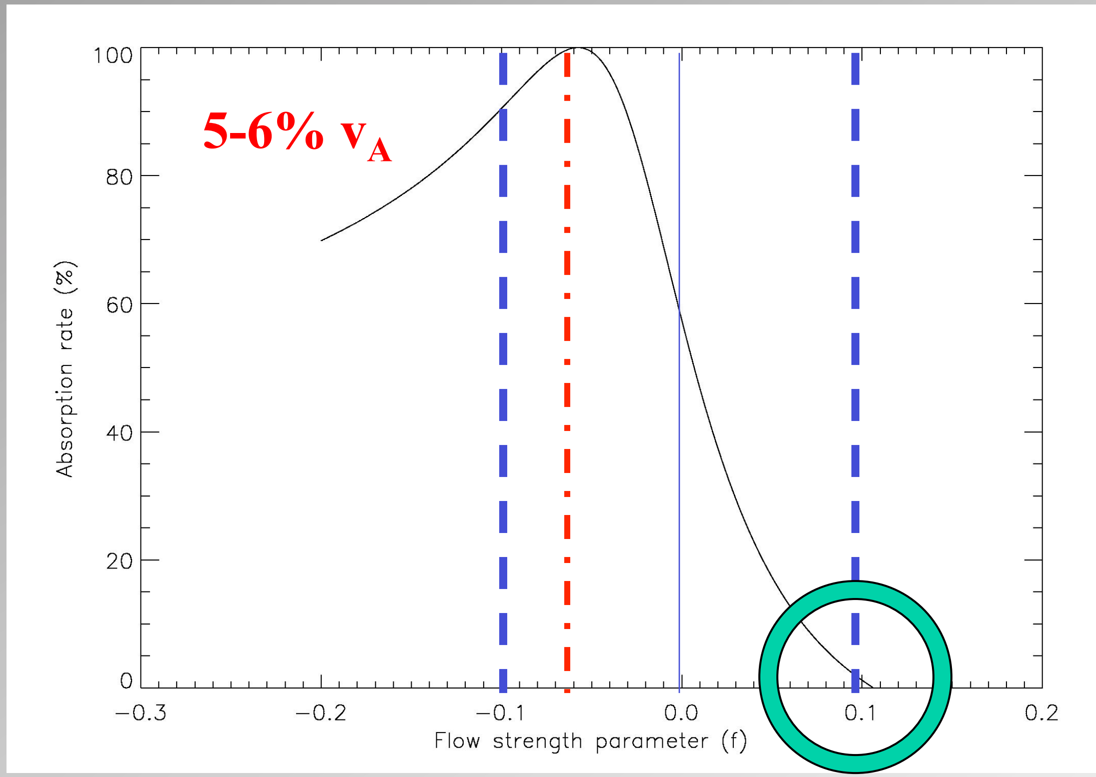
$$[\xi_r] = -i\pi \frac{g_B C_A}{\rho B^2 |\Delta|} \operatorname{sgn}(\Omega)$$

$$[P_1] = -i\pi \frac{2B_z T C_A}{\rho B^2 \tau_A |\Delta|} \operatorname{sgn}(\Omega)$$

$$C_A = const$$

Jumps are **independent** of dissipative coefficient

Internal background motion



- Steady large-scale flows (e.g., Doyle *et al*. 1997)
- Flow has a **major influence** on resonant absorption

… RF

Nonlinear resonant MHD waves in stratified plasmas

- Importance of nonlinearity ⇒ Ruderman *et al*. 1997ab, Ballai *et al*. 1998ab, Erdélyi *et al*. 2001
- Nonlinearity slightly reduces the efficiency of coupling
- Nonlinearity generates **mean flows**
- Nonlinear correction: $\pi^2/8$

Result: **nonlinearity does not seem to be important**

… RF

Double polytropic plasma

Collisionless plasma

Double adiabatic approach: empiric

Anisotropy: $\perp$ and $||$ temperature, pressure different

Resonant flow instability

Magnetosheath – magnetopause - magnetosphere

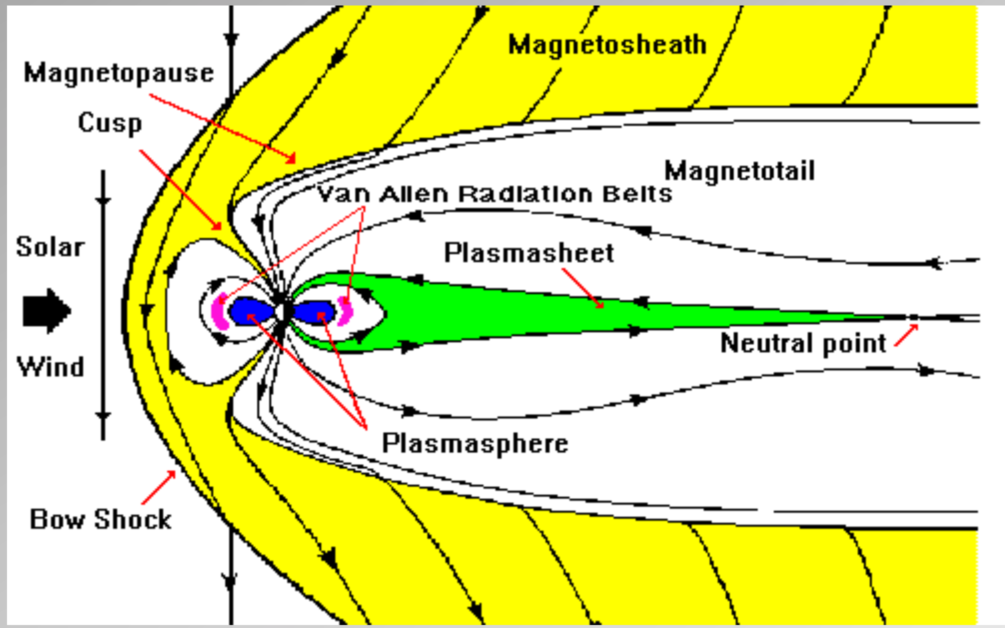
Heliopause – interstellar wind

Slow/fast solar wind boundary layer

Sunspots/Coronal plums

Helioseismology

Resonant flow instability



Magnetosheath – magnetopause – magnetosphere

Generation of Pc5 waves, energisation of magnetosphere

Resonant flow instability

Magnetosheath – magnetopause – magnetosphere

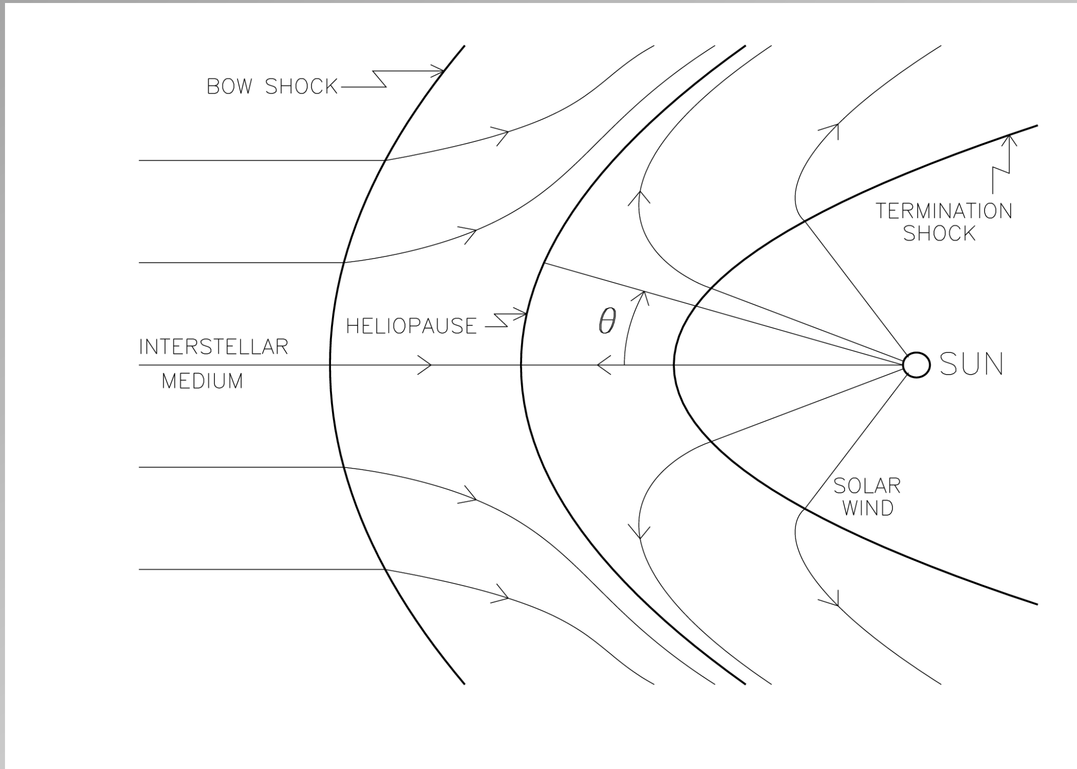
Heliopause – interstellar wind

Slow/fast solar wind boundary layer

Sunspots/Coronal plums

Helioseismology

Resonant flow instability



Heliopause – interstellar wind

Resonant flow instability

Magnetosheath – magnetopause – magnetosphere

Heliopause – interstellar wind

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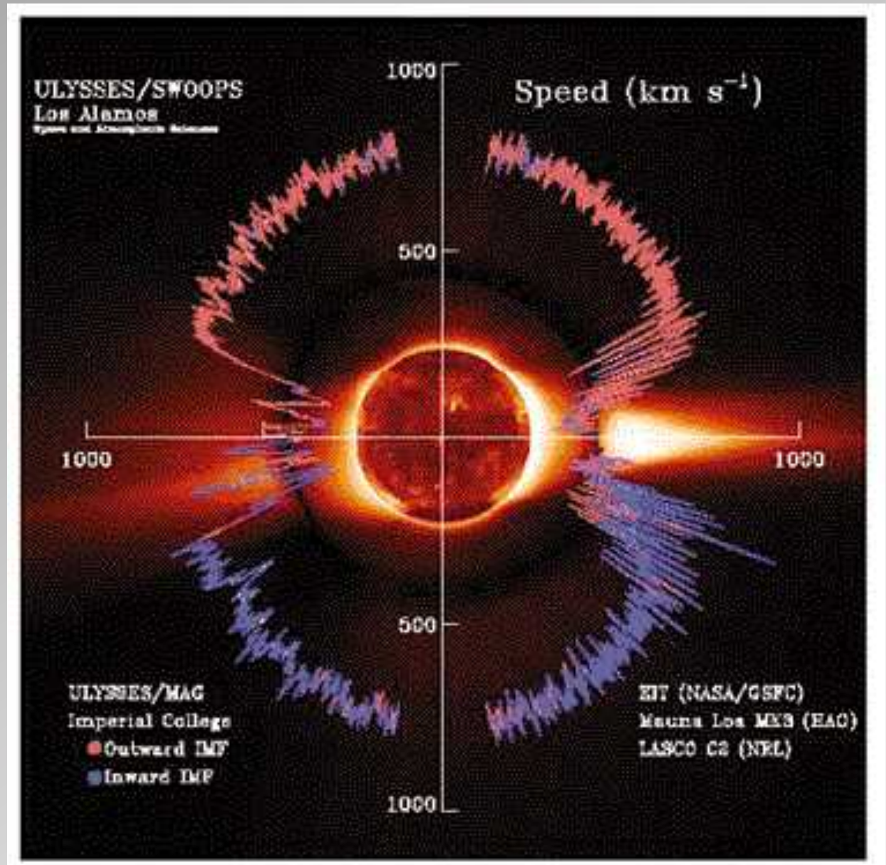

McComas, D.J., et al., Geophys. Res. Lett., 25, 1-4, 1998

Slow/fast solar wind

Resonant flow instability

Magnetosheath – magnetopause – magnetosphere

Heliopause – interstellar wind

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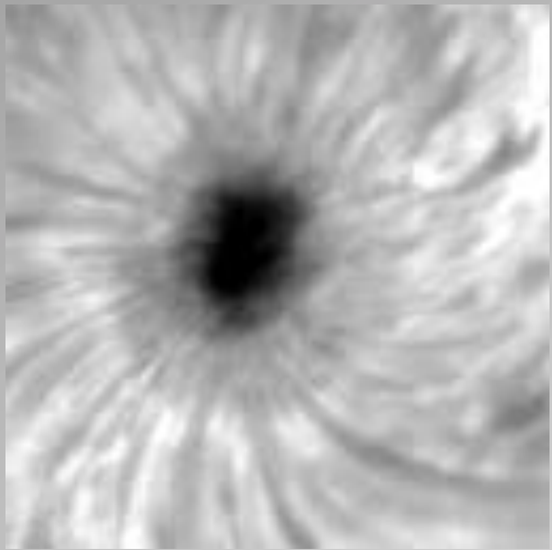
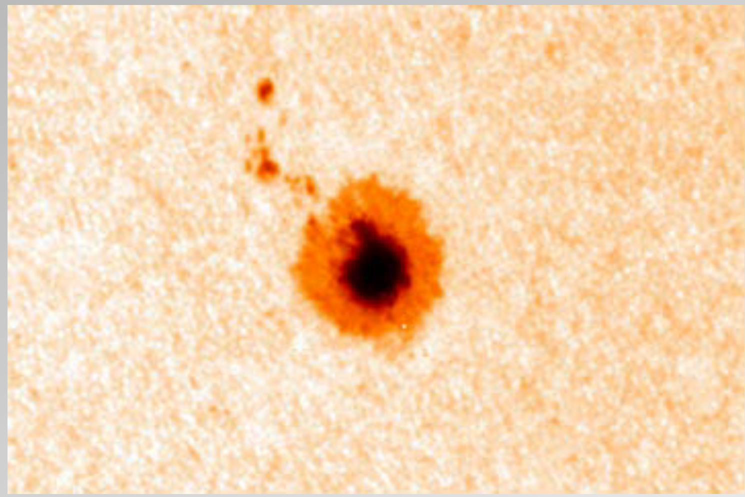
Helioseismology

Resonant flow instability





Running penumbral wave generation by RFI

Resonant flow instability

Magnetosheath – magnetopause – magnetosphere

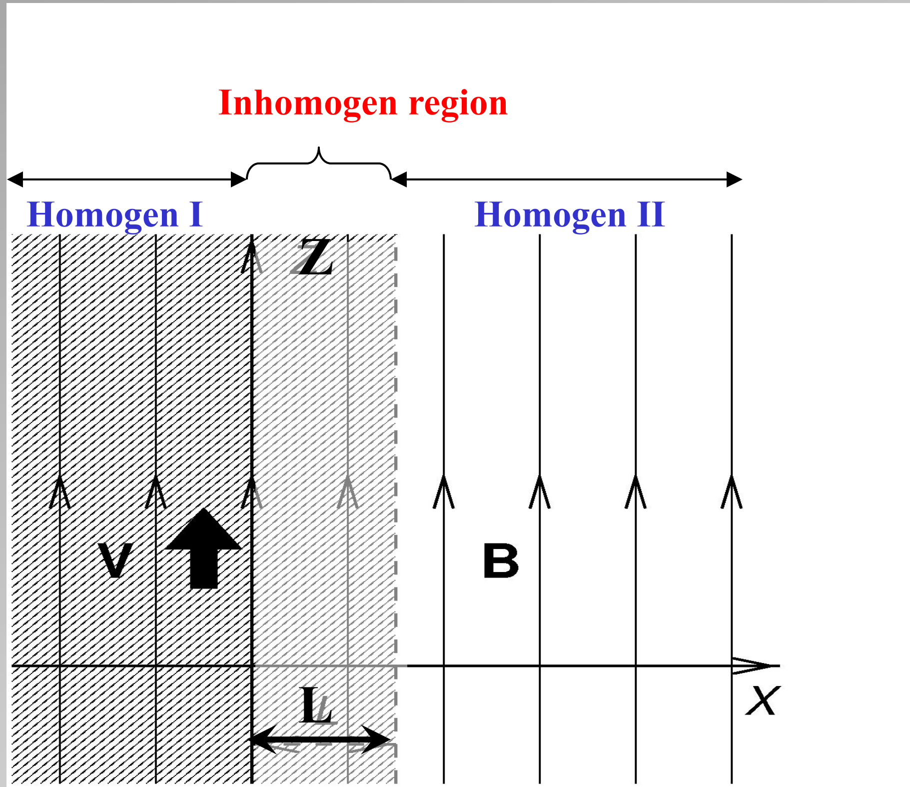
Heliopause – interstellar wind

Slow/fast solar wind boundary layer

Sunspots/Coronal plums

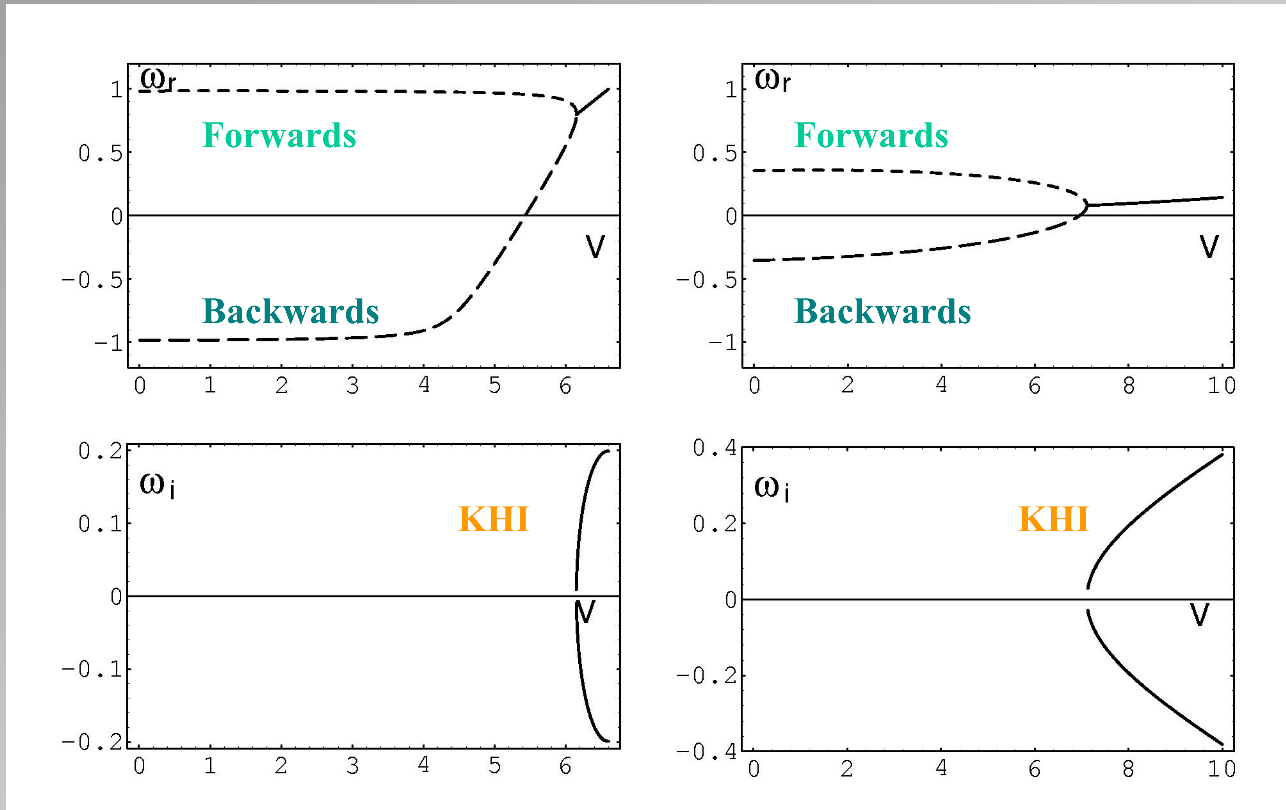
Helioseismology???

Model equilibrium state



Steady equilibrium state

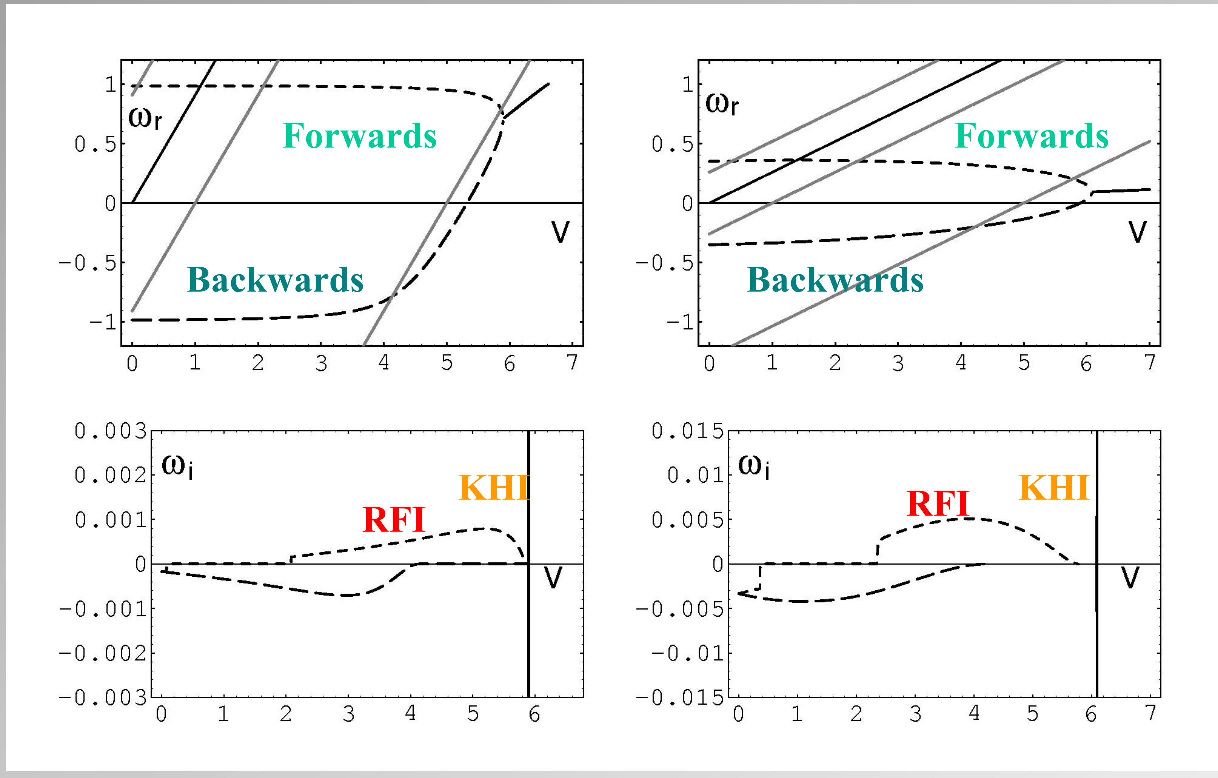
Kelvin-Helmholtz Instability



Fast modes (a, θ=25°; b, θ=75°); β=0 (cold plasma); L=0

Forwards & **backwards** propagation, **KHI**

Resonant Flow Instability (RFI)

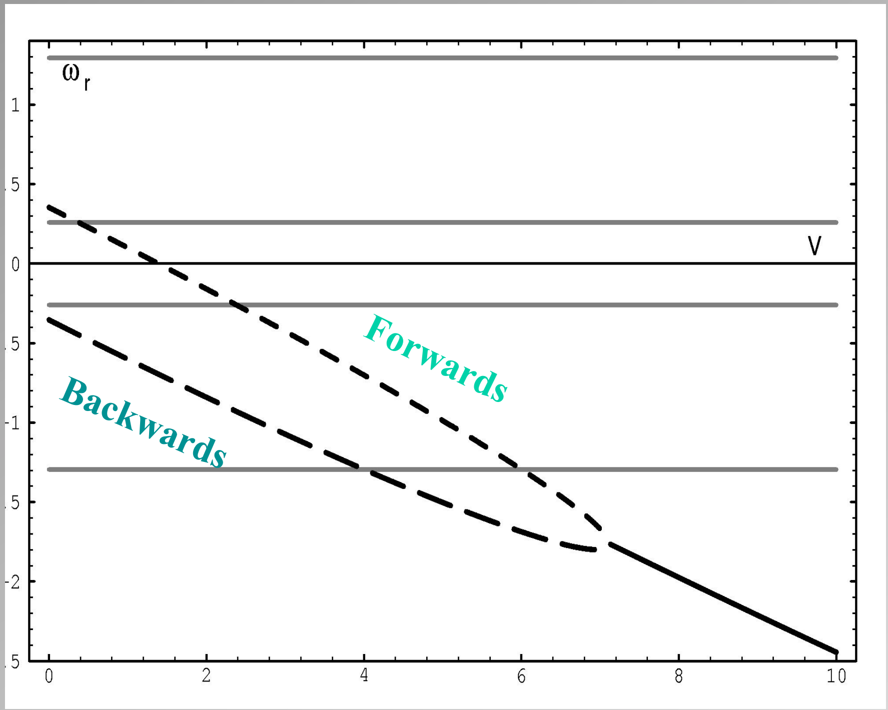


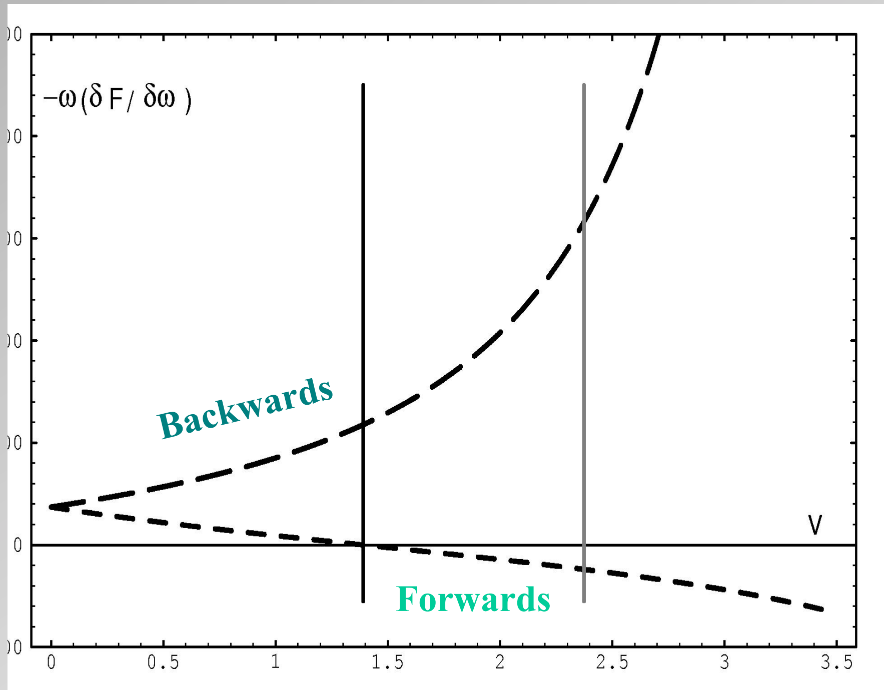
β=0; L=0.1 (non-uniform layer); Forwards & backward propagation;

Resonant flow instability < KHI

Hollweg *et al*. '90; Erdélyi & Goossens '96; Tirry *et al*. '98, Csík et al. '98, Andries *et al*, '00, 01, Taroyan & Erdélyi '02abc, etc.

Negative Energy Waves (NEW)





Non-zero ß

β=0.1; L=0.1

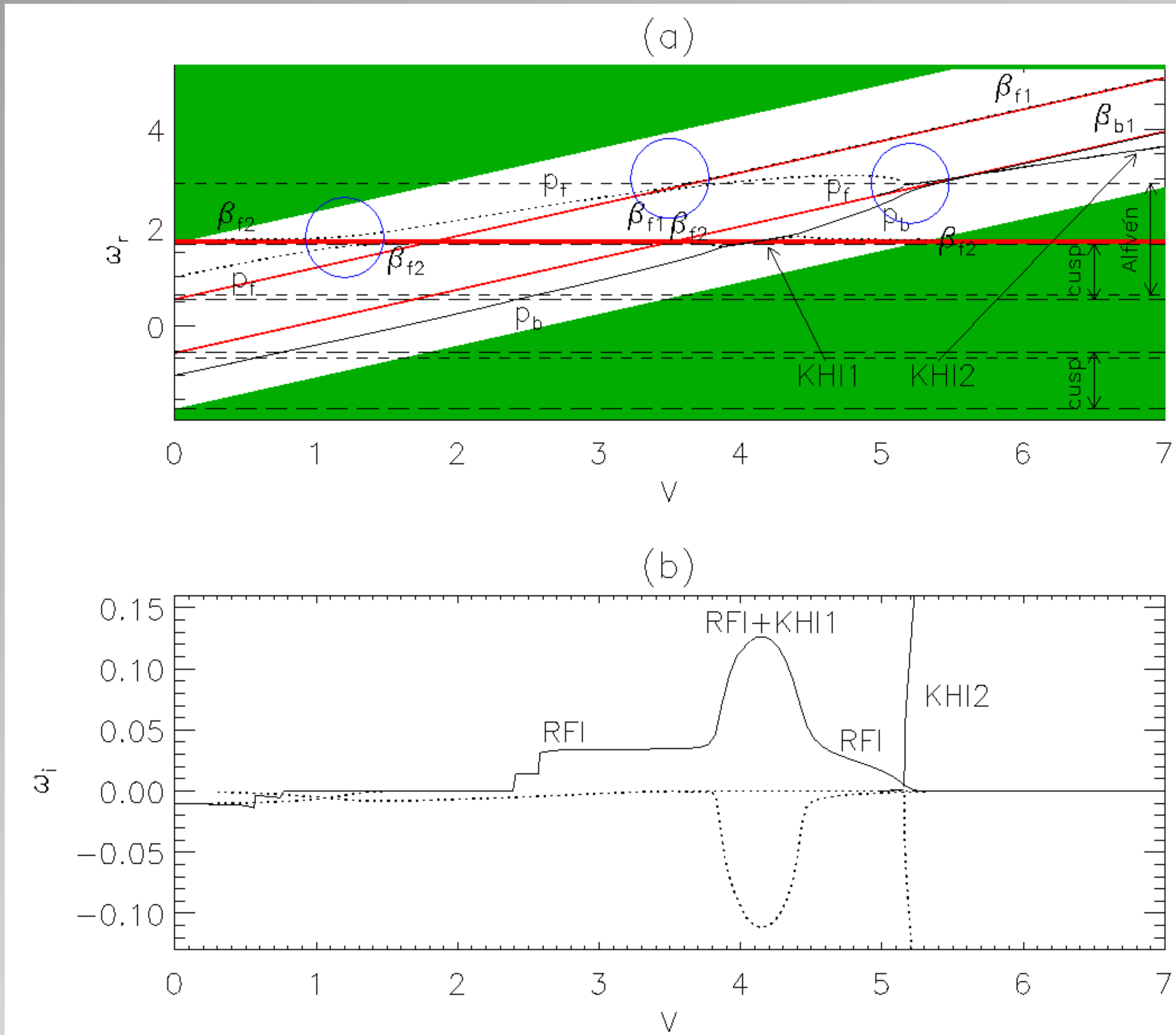
(non-uniform layer);

Two continua:

slow+Alfvén;

Two RFIs < KHI

Andries *et al*, '01; Erdélyi & Taroyan '02;

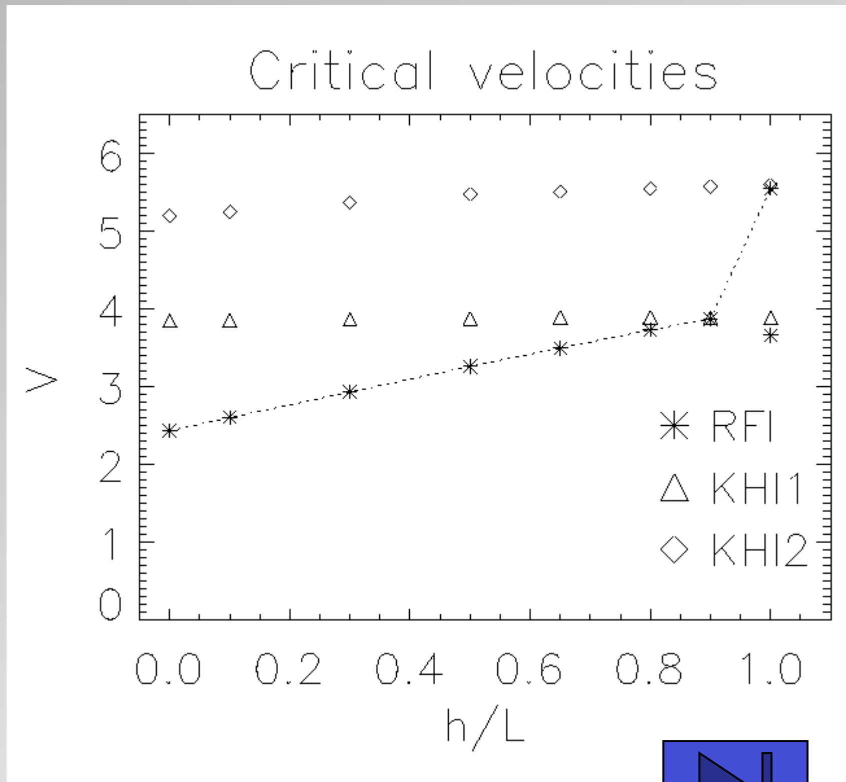


Variation of shear flow layer (h)

- Observations (Siscoe et al., 1994) show that the equilibrium plasma **flow changes continuously** from its constant value in the magnetosheath to small values in the magnetosphere.

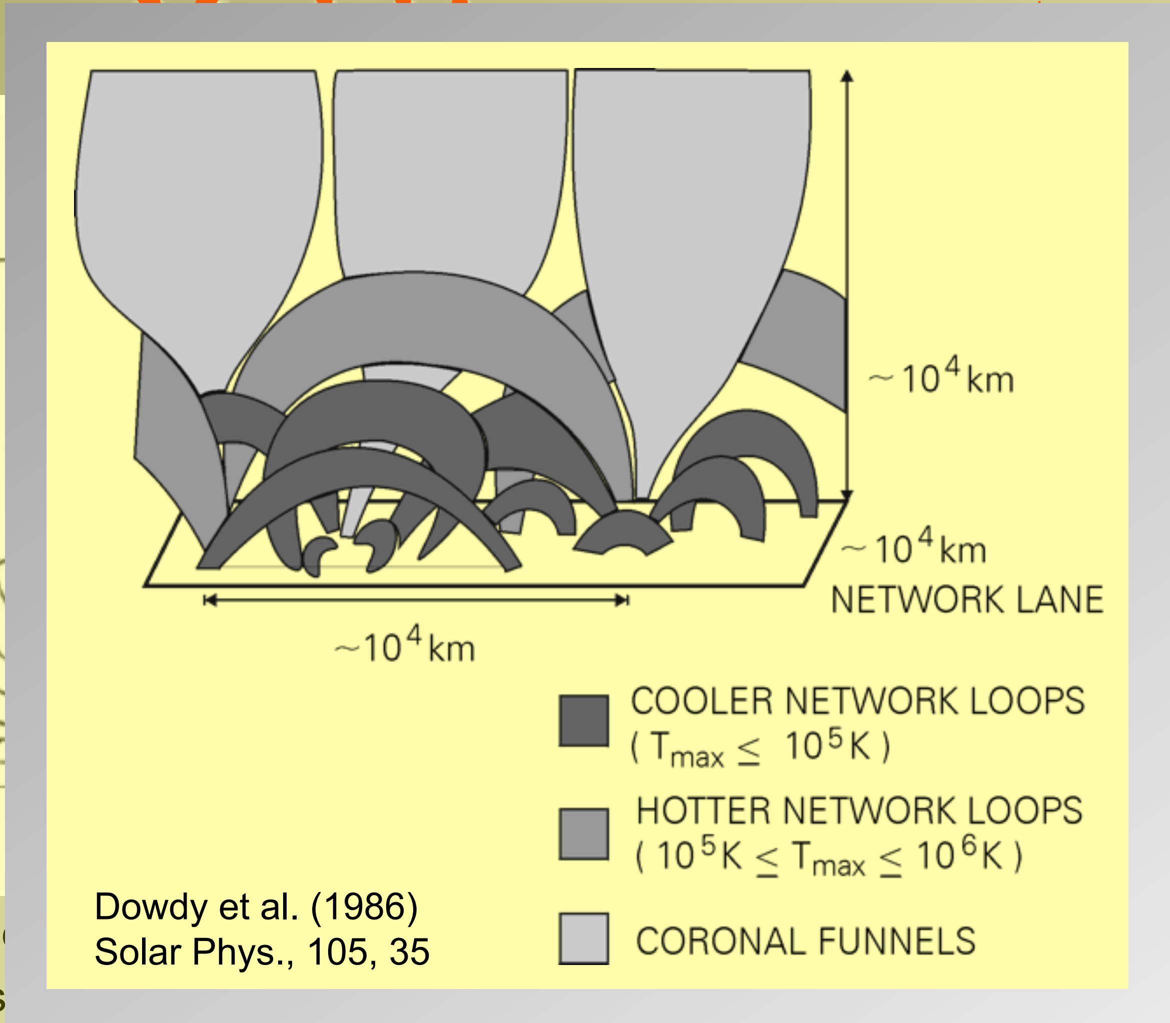
- The critical velocities for the **KHIs are not affected very much** by the introduction of the inhomogeneous layers.

- The **critical velocities** for the **RFI** tend to **increase** and the corresponding amplitudes decrease **with increasing h**.



What we really need?

- **Observations, observations, observation**
- Indirect observations (e.g. Erdélyi *et al.* ‘98)
- Direct observations of waves (Jess et al. ’09)
- Evidence for **resonant waves** (e.g. **mean flow**)
- **Observe reconnection driven (resonant) MHD waves**
- **Observe MHD wave driven reconnection**
- **JOPs/HOPs** + Ground-based (SST, DST/ROSA)

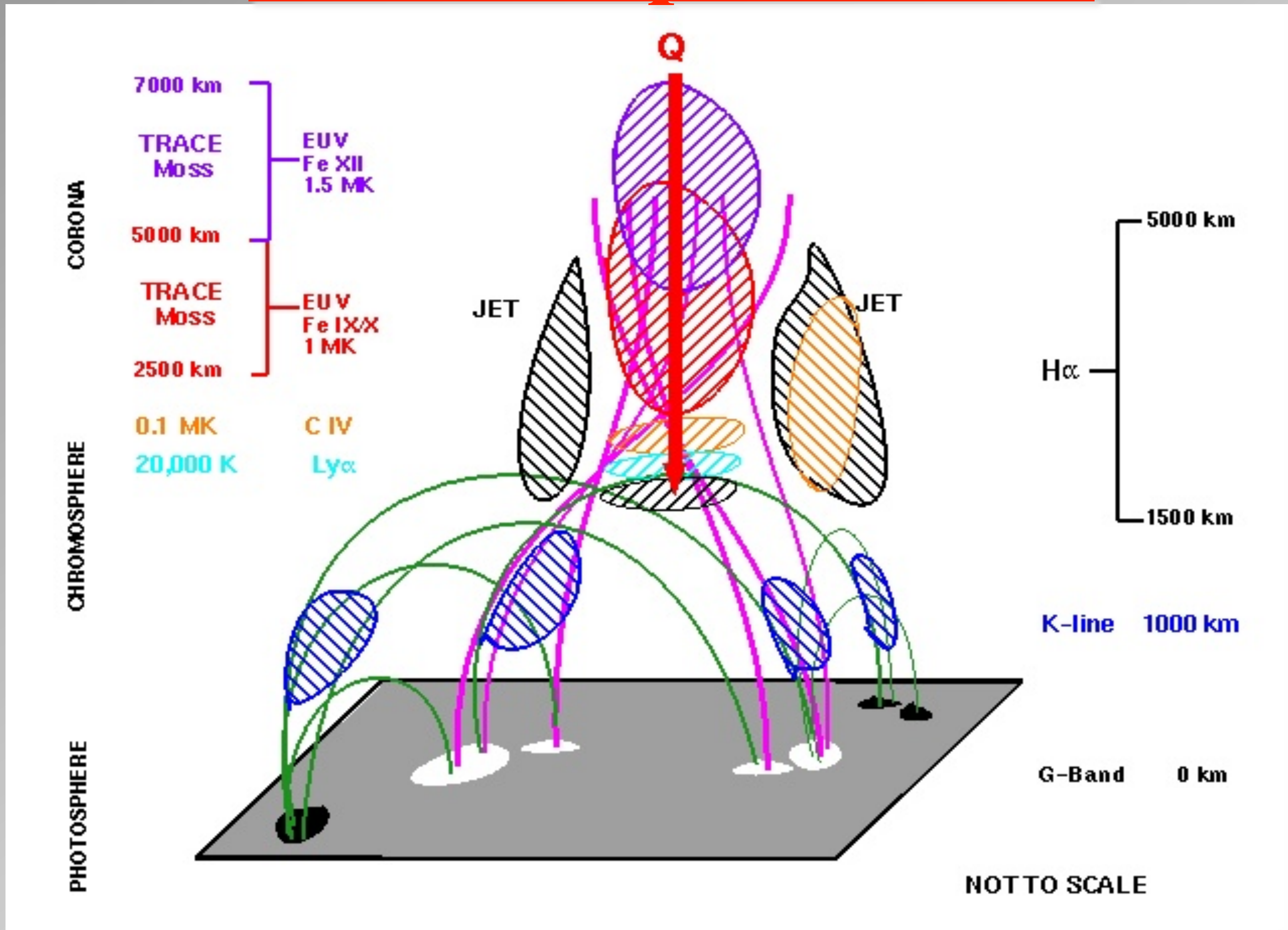


2D stati

➢ press

Gabriel (1976), Phil. Trans. A281, 339

Model improvement



(De Pontieu, Tarbell, Erdélyi, ApJ 590, 502, 2003)

Conclusions

- There are MHD waves in the Sun, in STP
- MHD waves are natural for plasma heating/acceleration
- MHD waves are sensitive to flows
- *Must* take into account effects of inhomogeneity/structuring. The structuring plays a crucial role in the wave dissipation and transformation.
- *The waves are an efficient tool for MHD seismology of the solar atmosphere*, which allows us to determine values of the mean parameters of the corona, such as the magnetic field strength, density, pressure, and transport coefficients. Some of these values: the magnetic field strength, viscosity, resistivity and thermal conductivity, are not open to measurement by any other means. We can do this by measuring the properties of MHD waves and oscillations (periods, wavelengths, amplitudes, temporal and spatial signatures), combined with theoretical modelling of the wave phenomena.

The end