The Solar Dynamo

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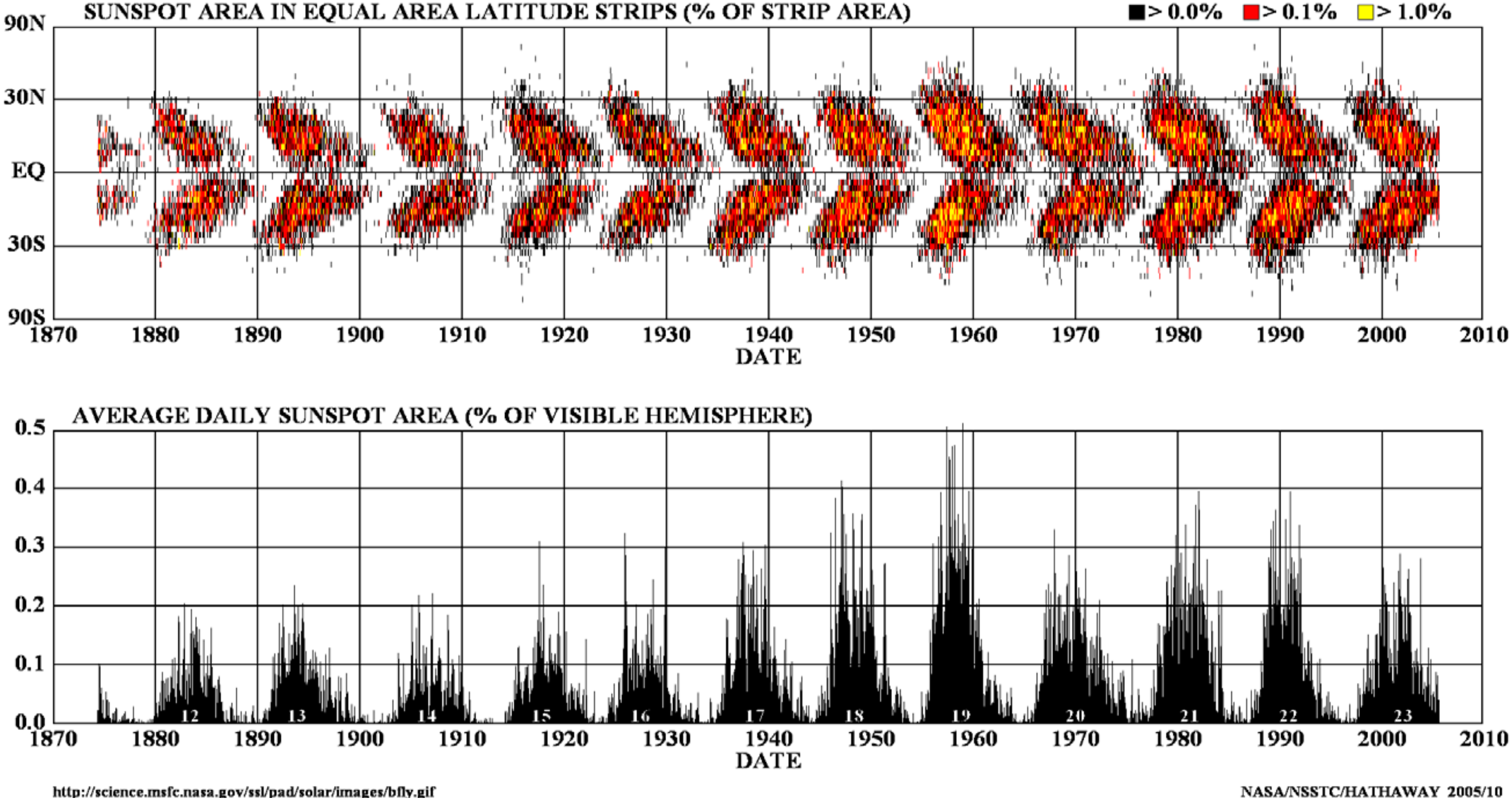
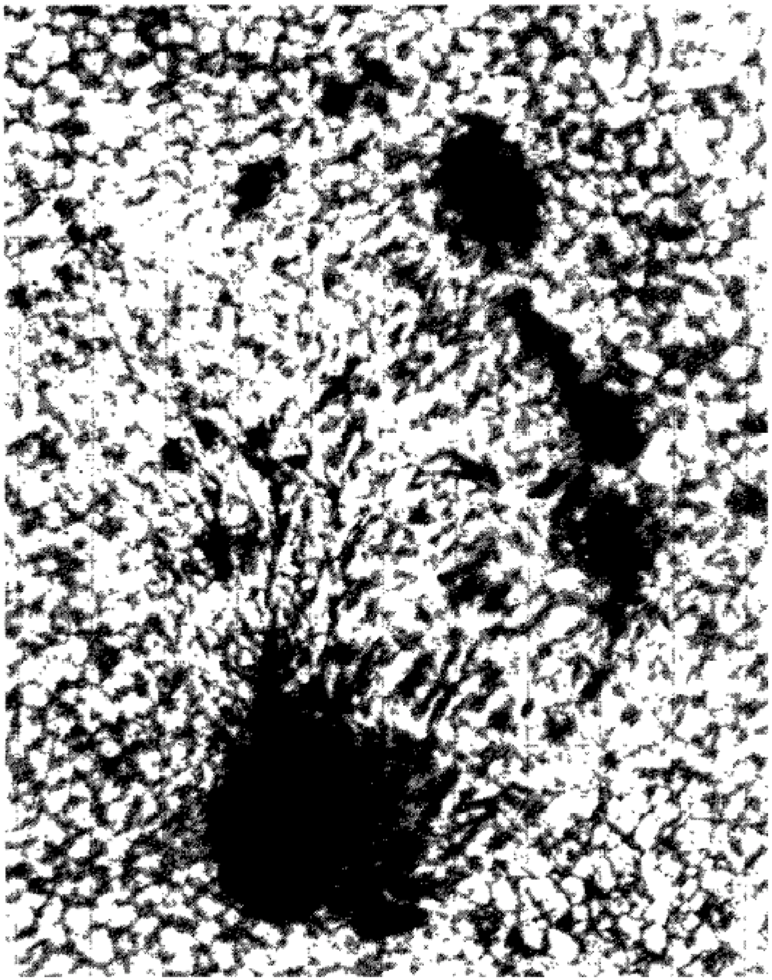
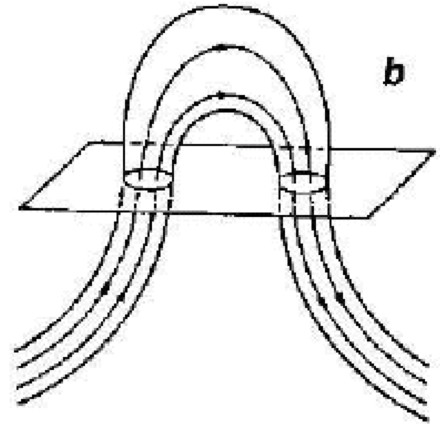
Brief history

- ~ 150 years ago – Schwabe discovered the 11- year cycle of sunspots (1844)
- 1858: Carrington discovers latitudinal drift
- Maunder invents butterfly diagram
- ~ 100 years ago – Hale discovered strong magnetic field in sunspots (B about 3000 G) (1908)
- ~ 50 years ago – Parker formulated dynamo theory for the origin of astronomical magnetic fields (1955)

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS





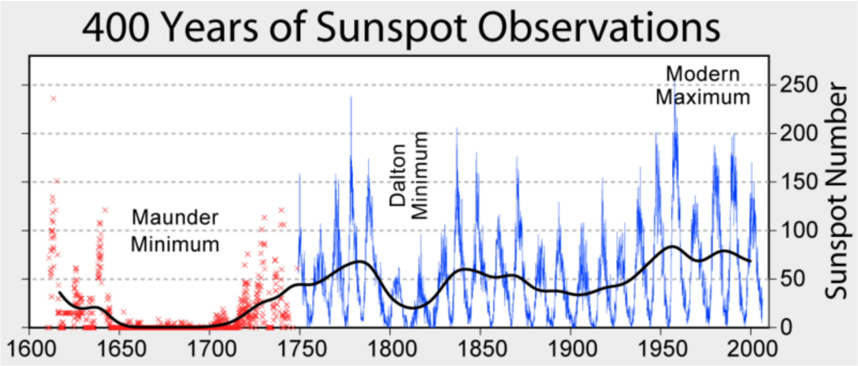


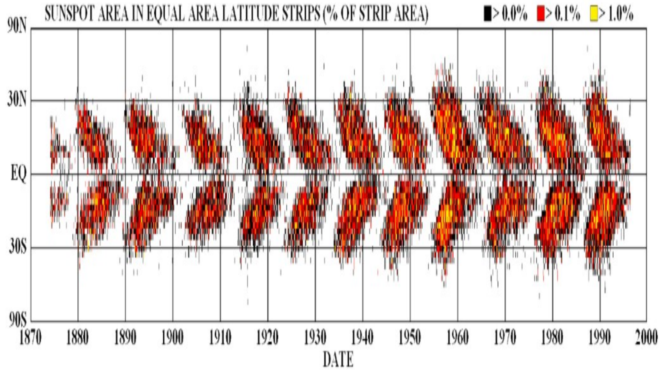
Hale et al. (1919) – Often two large sunspots are seen side by side with opposite polarities.

Sunspots are magnetic field concentrations in turbulent plasma

Origin of Solar Magnetic Field

- The origin of the solar magnetic field remains a stubborn challenge of astrophysics.
- At the solar surface the magnetic field assumes a complex, hierarchical structure in space and time.
- Systematic features such as the solar cycle and the buttery diagram point to the existence of a deep-rooted large-scale predominantly toroidal magnetic field.





Making a solar dynamo model

- Dynamo theory describes the process through which a rotating, convecting, and electrically conducting fluid acts to maintain a magnetic field.

- Dynamo theory of astrophysical bodies uses magnetohydrodynamic equations to investigate how the fluid can continuously regenerate the magnetic field.

The dynamo problem

Consists in finding /producing a (dynamically consistent) flow field that has inductive properties capable of sustaining B against Ohmic dissipation.

There are three requisites for the dynamo process to operate:

- An electrically conductive fluid medium
- Kinetic energy provided by body rotation
- An internal energy source to drive convective motions within the fluid.

Induction or creation of magnetic field is described by the induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \eta \nabla^2 \mathbf{B} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

In the interior of the Sun

the collisional mean-free path of microscopic constituents $<<$ competing plasma length scales

fluid motions are non-relativistic, the plasma is electrically neutral and non-degenerate.

$\rightarrow$ Ohm's law holds
Ampere's law in its pre-Maxwellian $\rightarrow$ MHD induction equation

The magnetohydrodynamical (MHD) induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \qquad (1)$$

where $\eta = c_2/4\pi\sigma_e$ is the magnetic diffusivity

the divergence-free condition $\nabla \cdot \mathrm{B} = 0$

An evolution equation for the flow field u:

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + 2\Omega \times \mathbf{u} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \frac{1}{4\pi\rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot \tau$$

The Navier–Stokes equations

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} + 2\Omega \times \mathbf{u} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \frac{1}{4\pi\rho}(\nabla \times \mathbf{B}) \times \mathbf{B} + \frac{1}{\rho}\nabla \cdot \tau.$$

where τ is the viscous stress tensor (2)

Complemented by:

$$\frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u})$$

conservation of mass and energy

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T + \epsilon$$

as well as an equation of state

Basic equations of solar magnetism

Solar convection zone governed by equations of compressible MHD

$$P = R\rho T \text{ (Perfect Gas)} \quad \frac{\partial \rho}{\partial t} = -\nabla \cdot (\rho \mathbf{u}) \text{(Continuity)}$$

$$\nabla \cdot \mathbf{B} = 0, \quad \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \text{ (Induction)}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{u}) = -\nabla \cdot (\rho \mathbf{u}\mathbf{u}) - \nabla P + \rho \mathbf{g} + \nabla \cdot \tau + \mathcal{F}_{other} \text{ (Momentum)}$$

$$(\frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla)\left(\frac{P}{\rho^{\gamma}}\right) = \text{Source and Loss terms (Energy)}$$

The Dynamo Problem

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Inductive action of the flow field + Resistive dissipation

- A dimensionless ratio of advection of magnetic field to diffusion. Magnetic Reynolds number:

$$\mathrm{Rm} = uL/\eta$$

Here η, u, and L are "typical" numerical values for the magnetic diffusivity, flow speed, and length scale over which B varies significantly.

Magnetic fields and flows

- Interaction of magnetic fields and flows due to induction (kinematics) and body forces (dynamics).
- Recall induction equation (from Faraday's Law, Ampère's Law and Ohm's Law)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}); \quad \eta = (\mu_0 \sigma)^{-1}$$

Induction *Dissipation*

- *Induction – leads to growth of energy through extension of field lines*
- *Dissipation – leads to decay of energy into heat through Ohmic loss.*

**Sufficiently vigorous flows convert mechanical into magnetic energy if Magnetic Reynolds number* $R_m \equiv \mathcal{UL}/\eta$ *is large enough*

In the solar cycle context

- Ultimately, amplification of B occurs by stretching the pre-existing magnetic field. This is readily seen upon rewriting the inductive term in Equation (1) as:

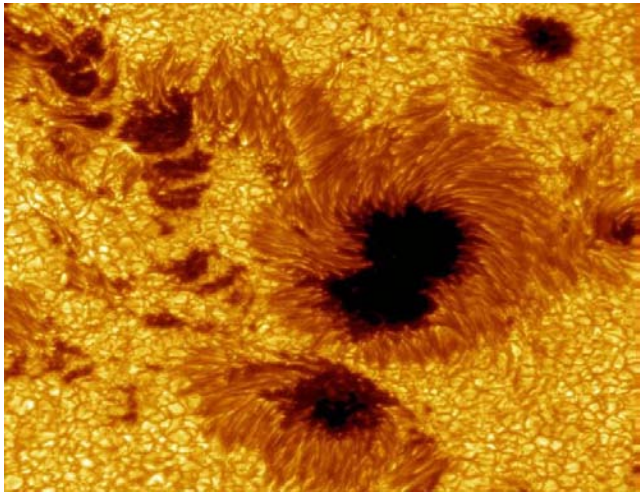
$$\nabla \times (\mathbf{u} \times \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{u} - (\mathbf{u} \cdot \nabla)\mathbf{B} - \mathbf{B}(\nabla \cdot \mathbf{u}).$$

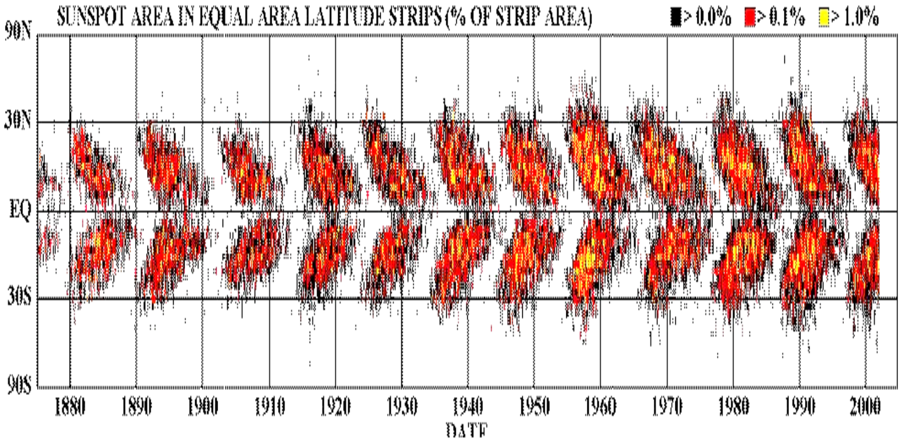
Exponential amplification of the magnetic field

The dynamo problem is reformulated towards to sustain the cyclic regeneration of the magnetic field associated with the observed solar cycle.

1) **Indicators of the Solar Cycle**

Sunspots





•Cyclical behaviour of the Sun: sunspots, observed since time of Galileo.
•Sunspots appear in pairs of opposite polarity, with leader spots of opposite polarity in the two hemispheres. (**Hale's Law**).
•Butterfly diagram shows a basica11y cycle, with long period modulation of cycles (Grand Minima) over times of order 200y. Also evidence of shorter modulation period (Glassberg cycle).

The Solar cycle is due to a large scale dynamo

Sun's natural decay time $\tau_\eta = R^2/\eta$ is very long (~10^{10} y) but cycle time is much less than τ_η :

Coherence of sunspot record suggests global mechanism operating at all longitudes.

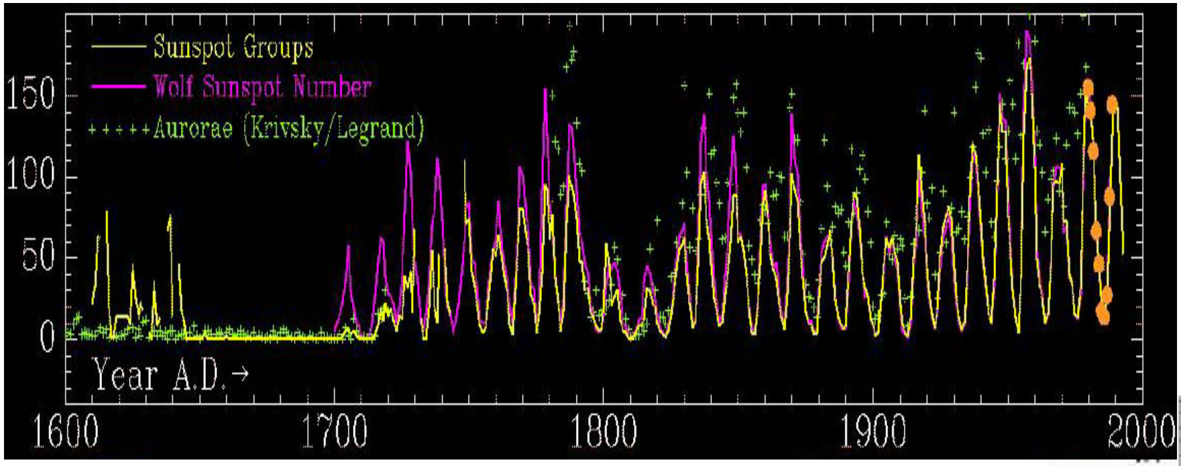
Polarity of leading spots and dipole moment changes every 11y.

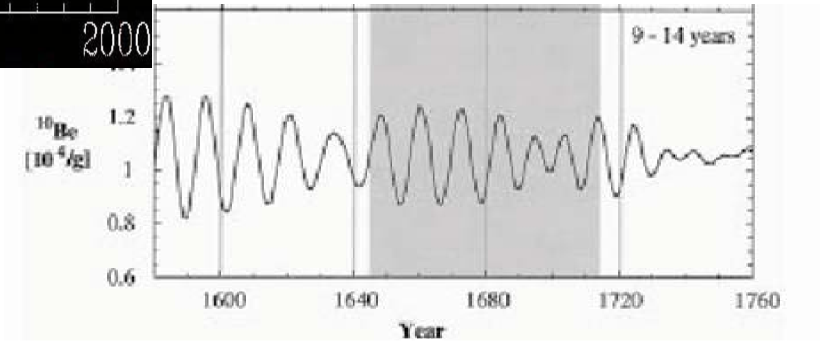
- Dynamo process in and/or just below convection zone. In this case velocity anomalies will be driven by Lorentz forces $\boldsymbol{j}\times\boldsymbol{B}$ and so have 11y period.

- Velocity data favours dynamo explanation. If there is a dynamo it must be fast

2) Modulations of the Cycle

- Grand minimum (hardly any sunspots: cold climate in N Europe ("little Ice Age") can be seen in early sunspot record. (Maunder Minimum).
- Proxy data provided by ^{14}C (tree rings) and ^{10}Be (ice cores). Intensities reflect cosmic ray abundance - varies inversely with global solar field. Shows regular modulations with period ~200y.
- Cyclic behaviour apparently persisted through Maunder minimum.
- Shorter modulation periods can be found (e.g. Glassberg 88y cycle)





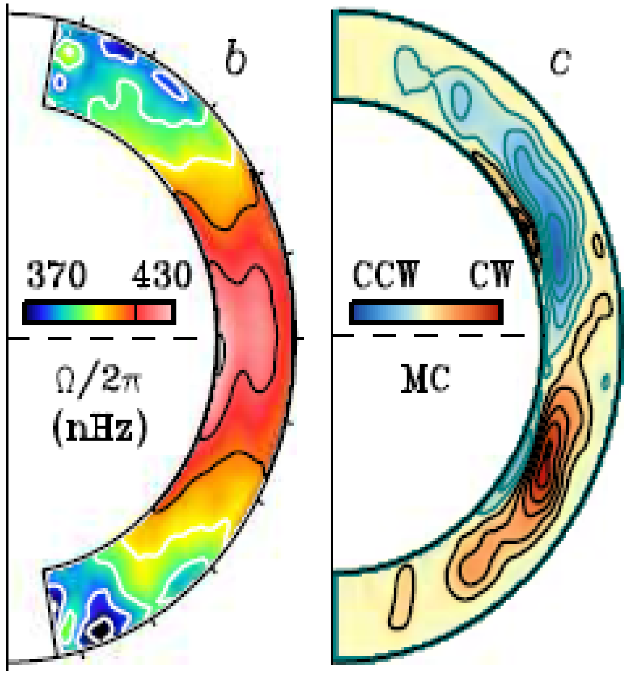
3) Helioseimology

Leighton, Noyes & Simon 1962 – discover solar oscillations.
Deubner 1974 – recognizes them as normal modes.

Moreover, helioseismology (Christensen-Dalsgaard, 2002) has now pinned down with good accuracy two important solar large-scale flow components:

•differential rotation throughout the interior,

•meridional circulation in the outer half of the solar convection zone (for reviews, see Gizon, 2004; Howe, 2009).



Miesch M S, Brun A S, DeRosa M L and Toomre J, 2008

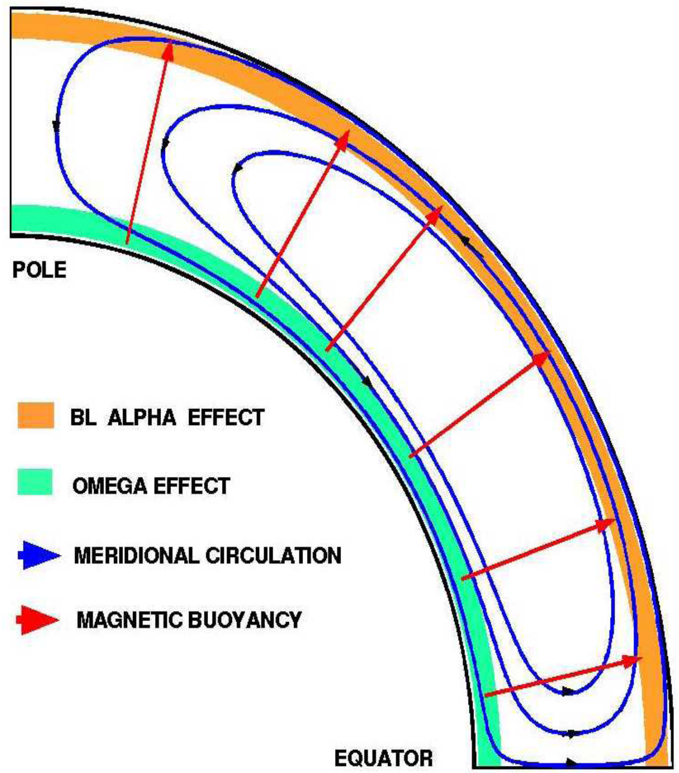
Flux transport dynamo in the Sun

(Choudhuri, Schussler & Dikpati 1995; Durney 1995)

Differential rotation

Babcock-Leighton process

Meridional circulation carries toroidal field equatorward & poloidal field poleward



Basic idea was given by Wang, Sheeley & Nash (1991)

Dynamo in the solar cycle

A model of the solar dynamo should also reproduce:

•cyclic polarity reversals with a $\sim$ 10 yr half-period,

•equatorward migration of the sunspot-generating deep toroidal field and its inferred strength,

•poleward migration of the diffuse surface field,

•observed phase lag between poloidal and toroidal components,

•polar field strength,

•observed antisymmetric parity,

•predominantly negative (positive) magnetic helicity in the Northern (Southern) solar hemisphere.

At the next level of “sophistication”

- Amplitude fluctuations, reproduce the many empirical correlations found in the sunspot record.
- Include an anticorrelation between cycle duration and amplitude (Waldmeier Rule)
- Alternation of higher-than-average and lower-than-average cycle amplitude (Gnevyshev–Ohl Rule)

- good phase locking,
- and occasional epochs of suppressed amplitude over many cycles (the so-called Grand Minima)

- torsional oscillations in the convective envelope

- The outer 30% in radius of the Sun are the seat of vigorous, thermally-driven turbulent convective fluid motions

- The solar dynamo problem is very hard to tackle as a direct numerical simulation of the full set of MHD equations

Most solar dynamo modelling work:

- relied on simplification of the MHD equations
- assumptions on the structure of the Sun’s magnetic field and internal flows.

I: Kinematic dynamo theory

A first drastic simplification of the MHD system of equations:

Velocity field is prescribed, instead of being a dynamic variable.

Induction equation becomes truly linear in B.

Using Maxwell´s equations simultaneously with the curl of Ohm´s Law

$\longrightarrow$

Magnetic field

$$\mathbf{u}(r,\theta) = \mathbf{u}_{\mathrm{p}}(r,\theta) + \varpi\Omega(r,\theta)\hat{\mathbf{e}}_{\phi}$$

- One arrives at a critical ***magnetic Reynolds number*** above which the flow strength is sufficient to amplify the imposed magnetic field, and below which it decays.

The most functional feature of kinematic dynamo theory is that it can be used to test whether a velocity field is or is not capable of dynamo action

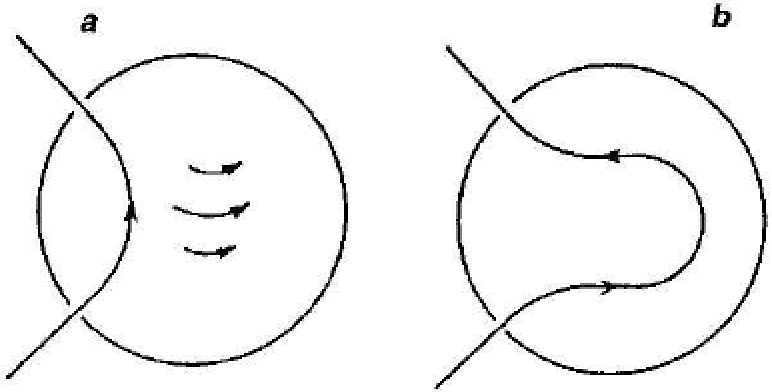
II: Axisymmetric magnetic field

$$\mathbf{B} = B_r \mathbf{e}_r + B_\theta \mathbf{e}_\theta + B_\phi \mathbf{e}_\phi$$

Poloidal Component
(responsible for weak fields)

Toroidal component
(gives rise to sunspots)

Toroidal field > bipolar
sunspots > poloidal field



Tachocline

Differential rotation produces
toroidal field from poloidal

- The sunspot butterfly diagram
- Hale's polarity law
- Synoptic magnetograms
- The shape of the solar corona at and around solar activity minimum.

Suggest a good first approximation:
The large-scale solar magnetic field is axisymmetric about the Sun's rotation axis, as well as antisymmetric about the equatorial plane.

Large-scale field

toroidal component (i.e., longitudinal)

+

poloidal component (i.e., contained in meridional planes)

$$\mathbf{B}(r,\theta,t) = \nabla \times (A(r,\theta,t)\hat{\mathbf{e}}_\phi) + B(r,\theta,t)\hat{\mathbf{e}}_\phi.$$

solenoidal constraint $\nabla \cdot \mathbf{B} = 0$

MHD induction equation $\longrightarrow$ Evolution equations for A and B (coupled)

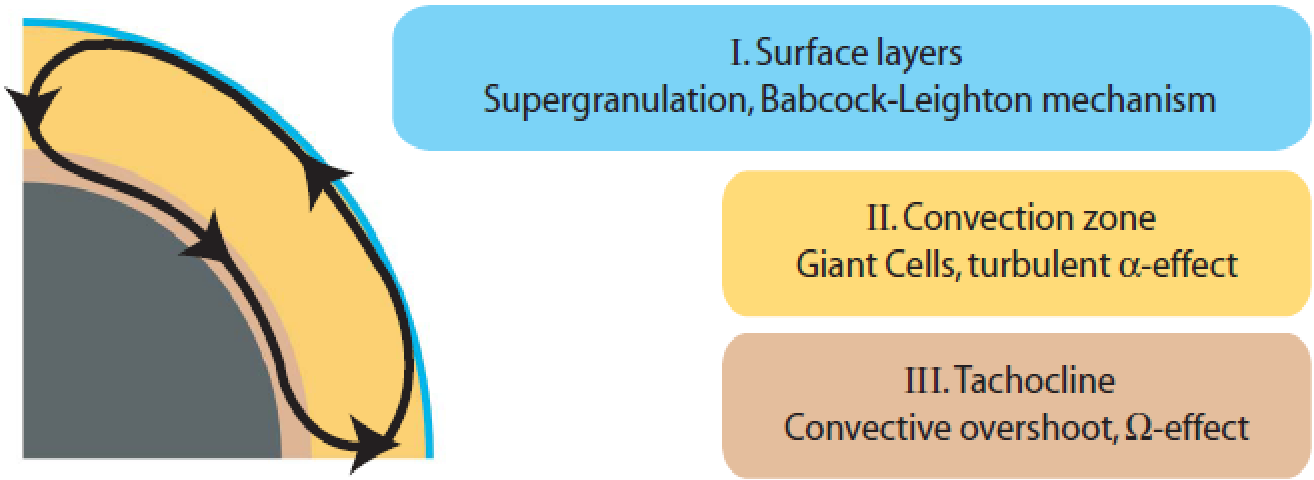
The φ-component with the Coulomb gauge $\nabla \cdot A = 0$ yields:

$$\frac{\partial A}{\partial t} + \frac{1}{s}(\boldsymbol{v}.\nabla)(sA) = \eta_{\mathrm{p}}\left(\nabla^2 - \frac{1}{s^2}\right)A + \alpha B,$$

$$\frac{\partial B}{\partial t} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_r B) + \frac{\partial}{\partial \theta}(v_\theta B)\right] = \eta_{\mathrm{t}}\left(\nabla^2 - \frac{1}{s^2}\right)B$$
$$+ s(\boldsymbol{B}_{\mathrm{p}}.\nabla)\Omega + \frac{1}{r}\frac{\mathrm{d}\eta_{\mathrm{t}}}{\mathrm{d}r}\frac{\partial}{\partial r}(rB)$$

The code Surya solves these
Equations, (Nandy & Choudhuri 2002)

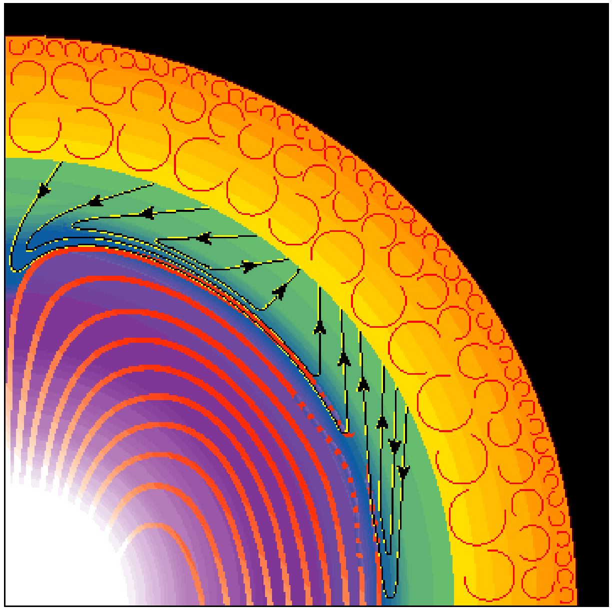
Dynamo models explicitly or implicitly divide the solar envelope into three distinct regions:



- The convection zone, region II, lies between 0.72R and 0.98R
- Region I then stretches from 0.98R to the photosphere.
- Region III includes the tachocline, r~ 0.68R-0.72R

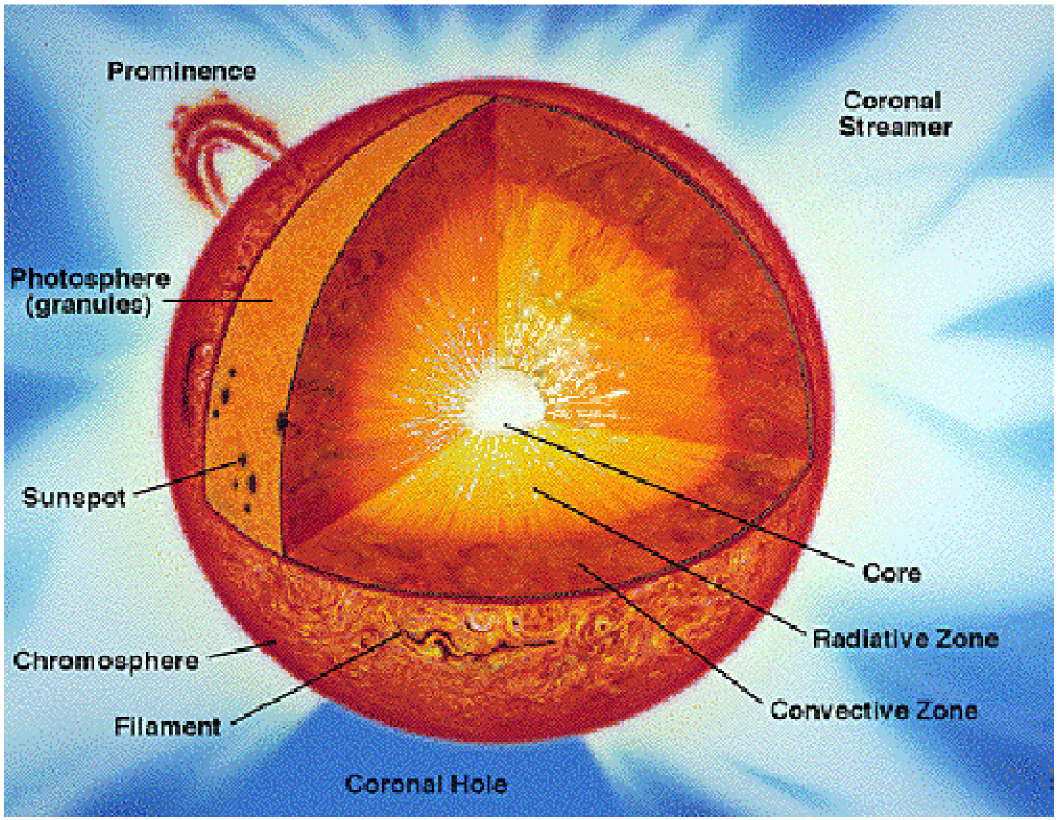
The Tachocline

The tachocline is the rotational shear layer uncovered by helioseismology immediately beneath the Sun's convective envelope, providing smooth matching between the latitudinal differential rotation of the envelope, and the rigidly rotating radiative **core**



- ***The solar tachocline is also situated near the base of the convection zone, and provides a mechanism to convert a weak poloidal magnetic field into a strong toroidal magnetic field***

The Solar interior and surface



Solar Interior

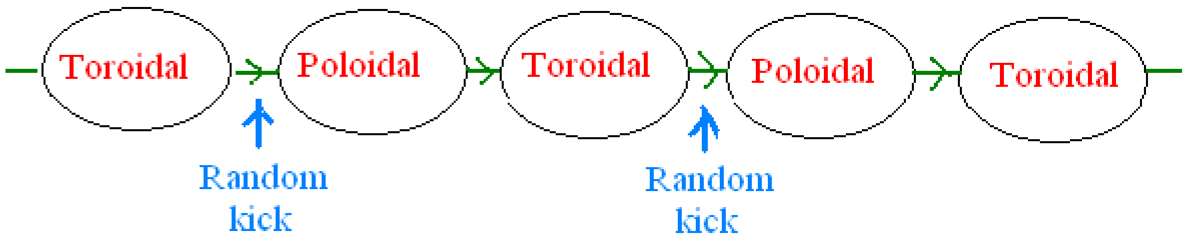
- Core
- Radiative Interior
- (Tachocline)
- Convection Zone

Visible Sun

- Photosphere
- Chomosphere
- Transition Region
- Corona
- (Solar Wind)

Magnetic Field Mechanisms

Cyclic regeneration of Sun´s large scale field:



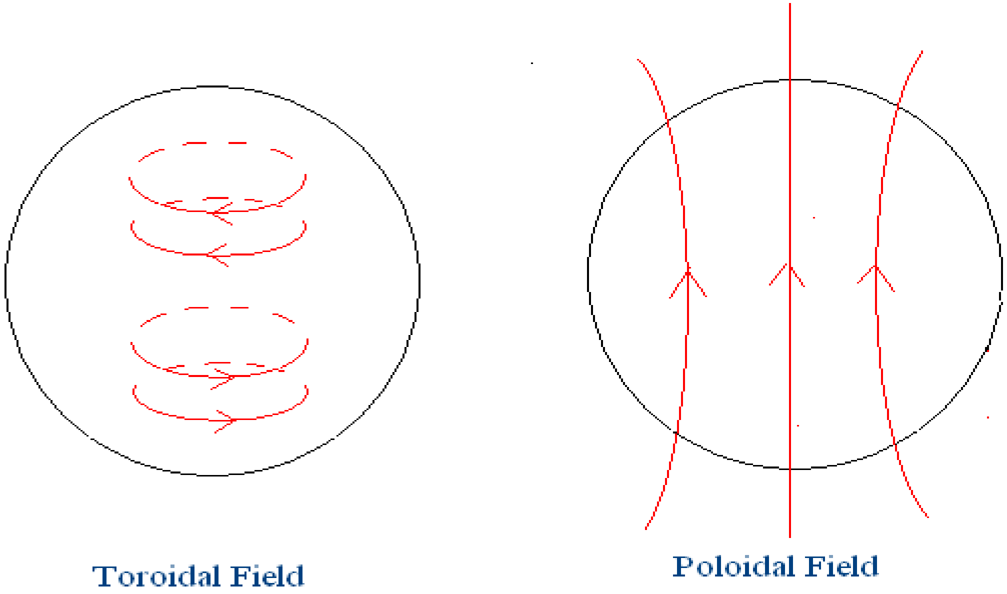
Choudhuri, Chatterjee &Jiang (2007)

T picks near Sunspot cycle maximun
P picks at the time of sunspots minimum

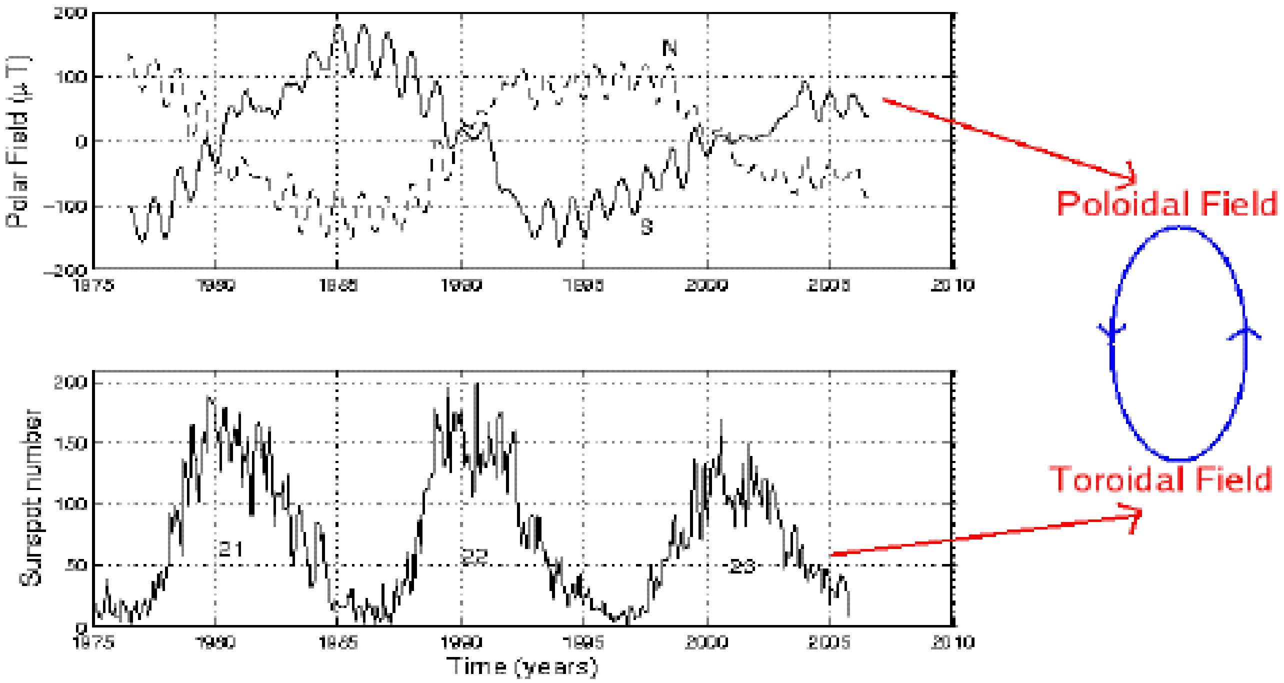
Polar field at the minimum gives an indication of the strength of the next solar maximum
(Schatten, Scherrer, Svalgaard &Wilcox 1978)

The dynamo problem can be broken into sub-problems:

a)Poloidal component →→ Toroidal
b)Toroidal component →→ Poloidal



Parker (1955) suggested oscillation between the toroidal and poloidal fields.



The polar fields and the sunspot number as functions of time

Poloidal to Toroidal

Large-scale flow field **u** as the sum of an axisymmetric azimuthal component (differential rotation), and an axisymmetric "poloidal" component $u_r(r,\theta)\hat{\mathbf{e}}_r + u_\theta(r,\theta)\hat{\mathbf{e}}_\theta$

$$\mathbf{u}(r,\theta) = \mathbf{u}_{\mathrm{p}}(r,\theta) + \varpi\Omega(r,\theta)\hat{\mathbf{e}}_\phi$$

$$\varpi = r\sin\theta$$

$$\frac{\partial A}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)A}_{\text{resistive decay}} - \underbrace{\frac{\mathbf{u}_{\mathrm{p}}}{\varpi}\cdot\nabla(\varpi A)}_{\text{advection}},$$

$P \rightarrow T$ production mechanism

↓

$$\frac{\partial B}{\partial t} = \underbrace{\eta\left(\nabla^2 - \frac{1}{\varpi^2}\right)B}_{\text{resistive decay}} + \underbrace{\frac{1}{\varpi}\frac{\partial(\varpi B)}{\partial r}\frac{\partial\eta}{\partial r}}_{\text{diamagnetic transport}} - \underbrace{\varpi\mathbf{u}_{\mathrm{p}}\cdot\nabla\left(\frac{B}{\varpi}\right)}_{\text{advection}} - \underbrace{B\nabla\cdot\mathbf{u}_{\mathrm{p}}}_{\text{compression}} + \underbrace{\varpi(\nabla\times(A\hat{\mathbf{e}}_\phi))\cdot\nabla\Omega}_{\text{shearing}}.$$

converting rotational kinetic energy into magnetic energy.

Toroidal to Poloidal

Additional source term is necessary?

There exist various mechanisms that can act as a source of poloidal field:

- **Turbulence and mean-field electrodynamics**
- **Hydrodynamical shear instabilities**
- **MHD instabilities**
- **The Babcock–Leighton mechanism**

Turbulent convection

- The outer ~ 30% of the Sun are in a state of thermally-driven turbulent convection.
- Anisotropic turbulence (gravity and Coriolis force)

Evolution of the largescale magnetic field on time scales longer than the turbulent time scale

- Mean-field electrodynamics offers a tractable alternative for turbulent MHD.

*mean components, $\langle u \rangle$ and $\langle B \rangle$, + small-scale fluctuating u', B'.

Upon this separation and averaging procedure, the MHD induction equation for the mean component becomes:

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times \left(\langle \mathbf{u} \rangle \times \langle \mathbf{B} \rangle + \langle \mathbf{u}' \times \mathbf{B}' \rangle - \eta \nabla \times \langle \mathbf{B} \rangle \right).$$

mean electromotive force $\mathcal{E}$

The next step is to express $\mathcal{E}$ in terms of the mean field ⟨B⟩

Expressing $\mathcal{E}$ as a truncated series expansion in ⟨B⟩ and its derivatives

$$\boldsymbol{\mathcal{E}} = \alpha : \langle \mathbf{B} \rangle + \beta : \nabla \times \langle \mathbf{B} \rangle$$

(Turbulent model is required for α and β)

The $\alpha\Omega$ dynamo equations

Adding the mean-electromotive force given by to the MHD induction equation leads:

T → P production

$$\frac{\partial\langle A\rangle}{\partial t} = \underbrace{(\eta+\eta_{\mathrm{T}})\left(\nabla^2 - \frac{1}{\varpi^2}\right)\langle A\rangle}_{\text{turbulent diffusion}} - \frac{\mathbf{u}_{\mathrm{p}}}{\varpi}\cdot\nabla(\varpi\langle A\rangle) + \underbrace{\alpha\langle B\rangle}_{\text{MFE source}},$$

$$\frac{\partial\langle B\rangle}{\partial t} = \underbrace{(\eta+\eta_{\mathrm{T}})\left(\nabla^2 - \frac{1}{\varpi^2}\right)\langle B\rangle}_{\text{turbulent diffusion}} + \underbrace{\frac{1}{\varpi}\frac{\partial\varpi\langle B\rangle}{\partial r}\frac{\partial(\eta+\eta_{\mathrm{T}})}{\partial r}}_{\text{turbulent diamagnetic transport}} - \varpi\mathbf{u}_{\mathrm{p}}\cdot\nabla\left(\frac{\langle B\rangle}{\varpi}\right) - \langle B\rangle\nabla\cdot\mathbf{u}_{\mathrm{p}}$$

$$+\underbrace{\varpi(\nabla\times(\langle A\rangle\hat{\mathbf{e}}_\phi))\cdot\nabla\Omega}_{\text{shearing}} + \underbrace{\nabla\times[\alpha\nabla\times(\langle A\rangle\hat{\mathbf{e}}_\phi)]}_{\text{MFE source}},$$

$P \rightarrow T$ production mechanism

The axisymmetric mean-field dynamo equations

Note the following:

Even if ⟨B⟩ is axisymmetric, the α-term in Equation will effectively introduce source terms in both the A and B equations, so that Cowling's theorem can be circumvented.

Parker's idea of helical twisting of toroidal fieldlines by the Coriolis force corresponds to a specific functional form for α, and so finds formal quantitative expression in mean-field electrodynamics.

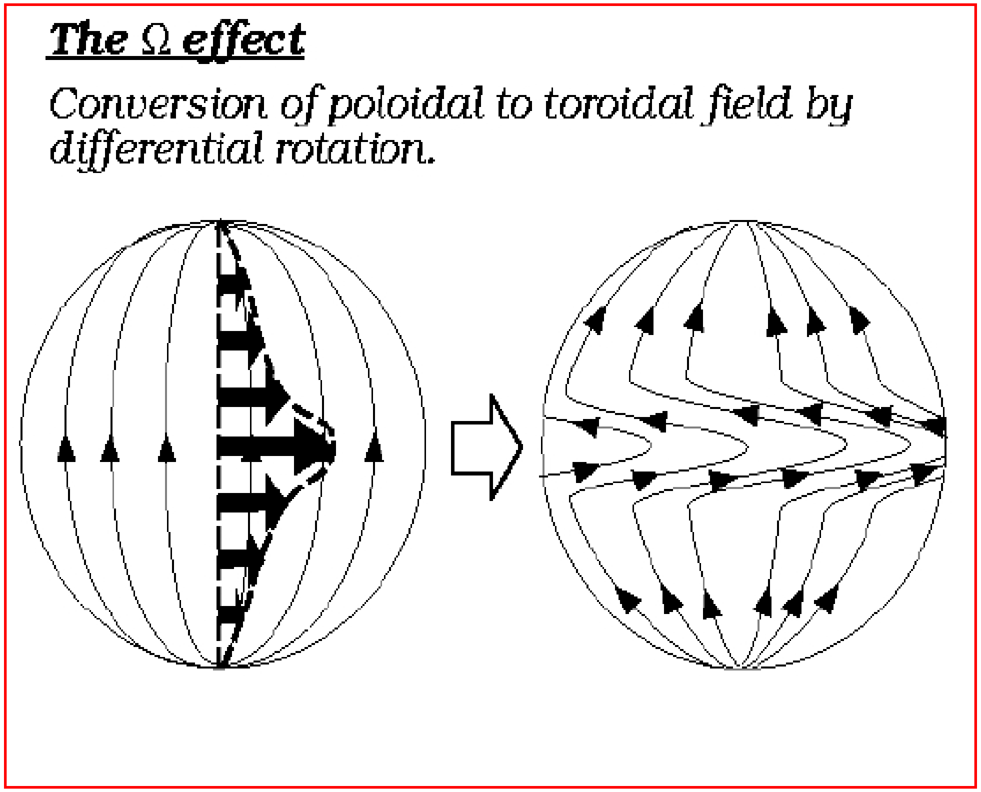
The production of a mean electromotive force proportional to the mean field is called the α-effect $\rightarrow$

Source of both A and B.

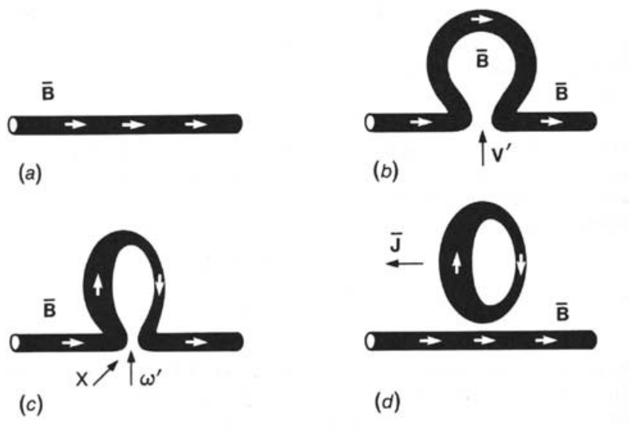
Viable $T \rightarrow P$ mechanism

The αΩ dynamo

Ω-effect



α-effect



Model ingredients

All kinematic solar dynamo models have:

(i)a solar structural model,

(ii)a differential rotation profile,

(iii)a magnetic diffusivity profile.

Analytic formulae for the angular velocity $\Omega(r, \theta)$ and magnetic diffusivity $\eta(r)$:

$$\frac{\Omega(r,\theta)}{\Omega_{\mathrm{E}}} = \Omega_{\mathrm{C}} + \frac{\Omega_{\mathrm{S}}(\theta) - \Omega_{\mathrm{C}}}{2}\left[1 + \mathrm{erf}\left(\frac{r - r_{\mathrm{c}}}{w}\right)\right]$$

$$\Omega_{\mathrm{S}}(\theta) = 1 - a_2 \cos^2\theta - a_4 \cos^4\theta,$$

$$\frac{\eta(r)}{\eta_{\mathrm{T}}} = \Delta\eta + \frac{1 - \Delta\eta}{2}\left[1 + \mathrm{erf}\left(\frac{r - r_{\mathrm{c}}}{w}\right)\right].$$

"turbulent" value ηT in the envelope to a much smaller diffusivity ηc in the convection-free radiative core $\Delta\eta = \eta c/\eta T$.

differential rotation profile:

$$\frac{\Omega(r,\theta)}{\Omega_{\rm E}} = \Omega_{\rm C} + \frac{\Omega_{\rm S}(\theta)-\Omega_{\rm C}}{2}\left[1+\mathrm{erf}\left(\frac{r-r_{\rm c}}{w}\right)\right]$$

$$\Omega_{\rm S}(\theta) = 1 - a_2\cos^2\theta - a_4\cos^4\theta,$$

$$\frac{\eta(r)}{\eta_{\rm T}} = \Delta\eta + \frac{1-\Delta\eta}{2}\left[1+\mathrm{erf}\left(\frac{r-r_{\rm c}}{w}\right)\right].$$

“turbulent” value η_T in the envelope to a much smaller diffusivity η_c in the convection-free radiative core $\Delta\eta = \eta_c/\eta_T$.

Calculating the α-effect and turbulent diffusivity

- Mean-field electrodynamics
- The task at hand is to calculate the components of the α and β tensor
- For an homogeneous, weakly, anisotropic turbulence:

$$\mathcal{E} = \alpha \langle \mathbf{B} \rangle - \eta_{\mathrm{T}} \nabla \times \langle \mathbf{B} \rangle .$$

form commonly used in solar dynamo modelling

In the kinematic regime (α and β are independent of the magnetic field fluctuations):

$$\alpha \sim -\frac{\tau_c}{3}\left\langle \mathbf{u}' \cdot \nabla \times \mathbf{u}' \right\rangle$$

$$\eta_T \sim \frac{\tau_c}{3}\left\langle \mathbf{u}' \cdot \mathbf{u}' \right\rangle ,$$

where τ_c is the correlation time of the turbulent motions.

Order-of-magnitude estimates: $\alpha \sim \Omega \ell$ and $\eta_T \sim v\ell$

At the base of the solar convection zone:

$$\alpha \sim 10^3 \text{ cm s}^{-1} \text{ and } \eta_T \sim 10^{12} \text{ cm}^2 \text{ s}^{-1}$$

In the literature:

$$10 - 10^3 \text{ cm s}^{-1} \text{ for } \alpha \text{ and } 10^{10} - 10^{13} \text{ cm}^2 \text{ s}^{-1} \text{ for } \eta_T.$$

Turbulent pumping

- In cases where the turbulence is more strongly inhomogeneous.
- An additional effect comes into play: **turbulent pumping**

$$\mathcal{E} = \boldsymbol{\alpha}^{S} : \langle \mathrm{B} \rangle + \boldsymbol{\gamma} \times \langle \mathrm{B} \rangle + \boldsymbol{\beta} : \nabla \times \langle \mathrm{B} \rangle .$$

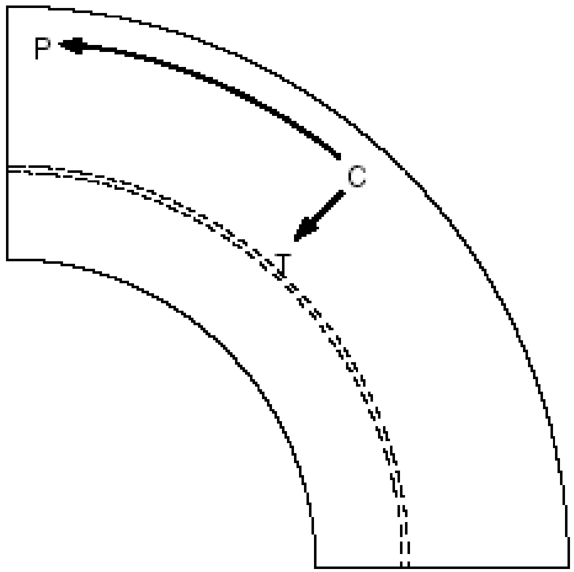
Leaving the kinematic regime, it is expected that both α and η_T should depend on the strength of the magnetic field

$$\alpha \sim -\frac{\tau_c}{3}\left[\langle \mathbf{u}' \cdot \nabla \times \mathbf{u}' \rangle - \langle \mathbf{a}' \cdot \nabla \times \mathbf{a}' \rangle\right], \quad \mathbf{a}' = \mathbf{B}'/\sqrt{4\pi\rho} \text{ is the Alfvén speed}$$

$$\alpha \to \alpha(\langle \mathbf{B} \rangle) = \frac{\alpha_0}{1 + (\langle \mathbf{B} \rangle / B_{\mathrm{eq}})^2}.$$

It remains an extreme oversimplification of the complex interaction between flow and field that characterizes MHD turbulence, but its wide usage in solar dynamo modeling.

Babcock-Leighton mechanism



Poloidal field produced here by Babcock-Leighton mechanism

- Joy's law: Bipolar sunspots have tilts increasing with latitude (D'Silva &Choudhuri 1993)
- Their decay produces poloidal field (Babcock 1961; Leighton 1969)

Future development

- Mean field models have led to qualitative understanding, but detailed calculation of α etc. in dynamic regime is controversial, and getting more so!
- Broad elements of basic processes (tachocline, pumping, buoyancy…) understood but more detailed calculations needed before useful quantitative information obtained.
- Better observations of cyclic behaviour in other stars with convective envelopes will help calibrate theories.
- A full-scale numerical model incorporating all relevant physics is a long way off; in the medium term any successful model will work by ‘wiring together’ detailed studies of the different regions.

CONCLUSIONS

Solar cycles are produced by a flux transport dynamo involving the following processes:

- Toroidal field generation in tachocline by differential rotation.
- Poloidal field generation at surface by Babcock- Leighton mechanism.
- Advection by meridional circulation.

Irregularities in cycles are primarily caused by fluctuations in the Babcock-Leighton process

CONCLUSIONS

- Remaining uncertainties about the nature of the deep-seated magnetic field and the alpha effect have thus far prevented the formulation of a coherent model for the solar dynamo.

Solar dynamo theory remains a vibrant field, fueled by fresh insights from helioseismology and increasingly sophisticated numerical simulations.