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Solar interior

ISWI-Europe Summer School
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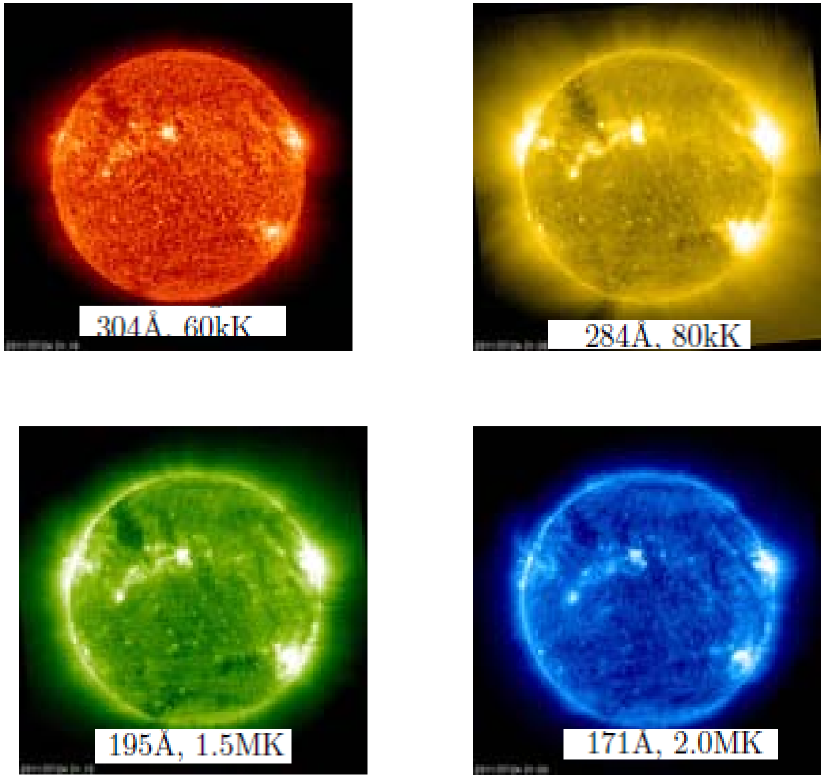
Two pictures of the Sun

Solar Physics



magnetic field rules

Theory of stellar evolution

gas pressure rules,
except in the sunspots

$$\left(\frac{R_e - R_p}{R_p}\right) \approx 3.5 \times 10^{-5}$$

Overview

1. Models of the Sun’s interior
2. Oscillations and seismic sounding
3. Internal rotation
4. Problems and prospects

Solar models 1960-2011

Stellar model $r, \quad M_r, \quad L_r, \quad p, \quad \rho, \quad T, \quad \boldsymbol{X}$

Assumptions & simplifications

Hydrostatic equilibfium, rotation & magnetic field ignored

$$\frac{dp}{dM_r} = -\frac{GM_r}{4\pi r^4} \qquad \frac{dr}{dM_r} = \frac{1}{4\pi r^2 \rho}$$

Energy transport

$$\frac{dT}{dM_r} = \frac{T}{p}\frac{dp}{dM_r} \times \begin{cases} \nabla_{\mathrm{rad}} & \text{if } \nabla_{\mathrm{rad}} \leq \nabla_{\mathrm{ad}} \\ \nabla_{\mathrm{ad}} + \nabla_n & \text{if } \nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}} \end{cases}$$

Heat budget

$$\frac{dL_r}{dM_r} = \epsilon - T\frac{dS}{dt}$$

fractional element abundance

$$X_1 \equiv X \qquad X_4 \equiv Y$$

$$\sum_{k=7} X_k \equiv Z = 1 - Z - Y$$

statistical & atomic physics

$$p(\rho, T, \boldsymbol{X})$$

$$S(\rho, T, \boldsymbol{X})$$

$$\nabla_{ad} \equiv \left(\frac{\partial \ln T}{\partial \ln p}\right)_{S, \boldsymbol{X}}$$

$$\nabla_{rad} = \frac{3\kappa L_r p}{16\pi G a c M_r T^4}.$$

$$\kappa(\rho, T, \boldsymbol{X})$$

convection theory

$$\nabla_n(\alpha_c) \equiv \nabla - \nabla_{\mathrm{ad}}$$

nuclear physics

$$\epsilon(\rho, T, \boldsymbol{X})$$

Chemical evolution

Uniform element distribution at zero age

nuclear physics

$$\frac{dX_i}{dt} = \mathcal{C}_{i,\mathrm{nuc}} + \mathcal{C}_{i,\mathrm{mac}} + \mathcal{C}_{i,\mathrm{mic}}$$

$\mathcal{C}_{i,\mathrm{nuc}}(\rho, T, \boldsymbol{X})$

$\mathcal{C}_{i,\mathrm{mic}}$ element diffusion included in 1990s

$\mathcal{C}_{i,\mathrm{mac}}$ standard $\nabla_{\mathrm{rad}} > \nabla_{\mathrm{ad}}$ complete mixing $\nabla_{\mathrm{rad}} \leq \nabla_{\mathrm{ad}}$ no mixing

non standard : overshooting, rotation induced mixing

coupling to angular momentum evolution

current age : 4.6 Gy

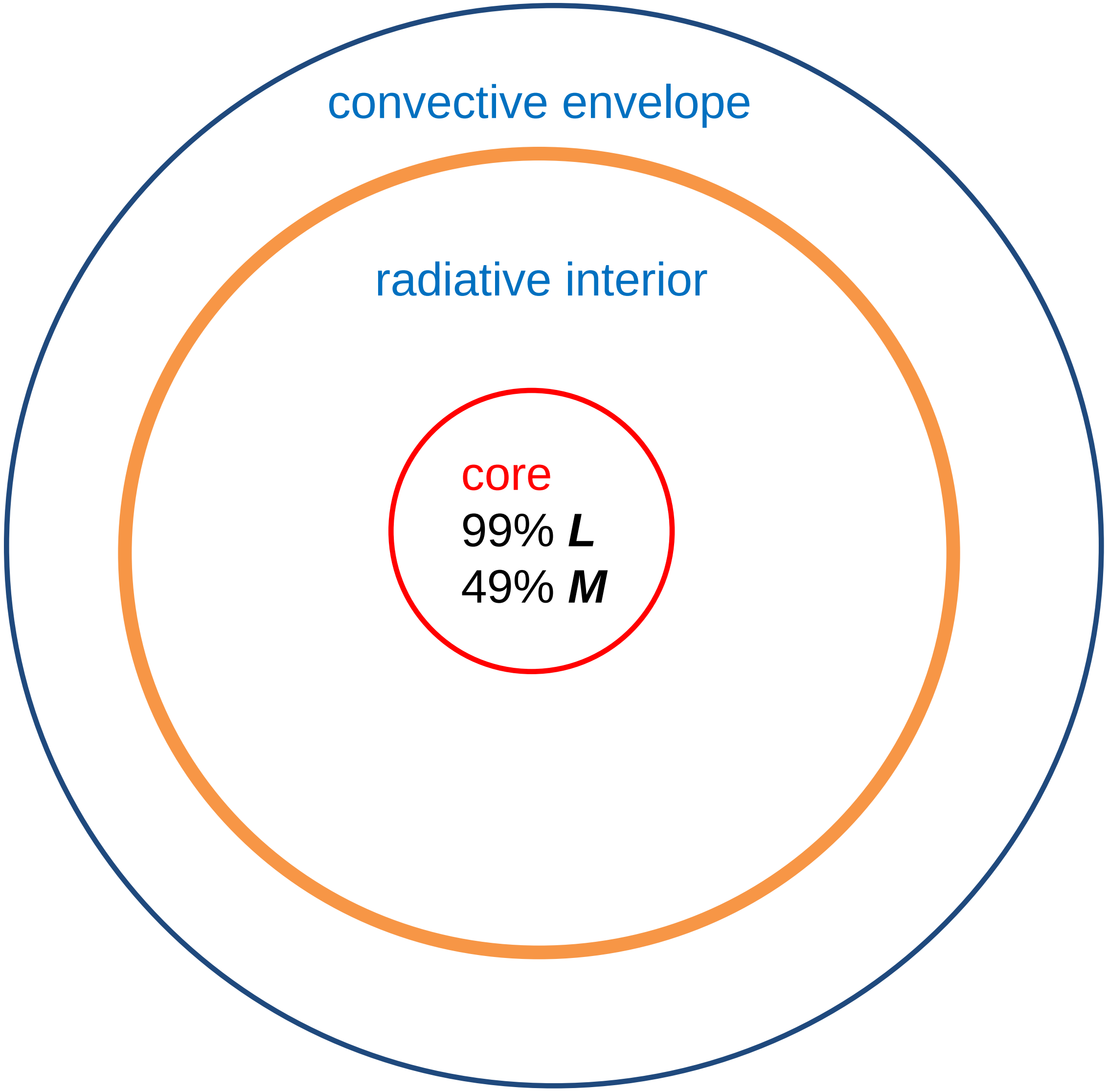
The standard construction

input data: : M, $(Z/X)_0$, **age**

adjust Y, α_c: to fit: L, R, Z/X using a standard stellar evolution code

Measured neutrino flux and oscillation frequencies used for testing

Schematic structure



convective envelope

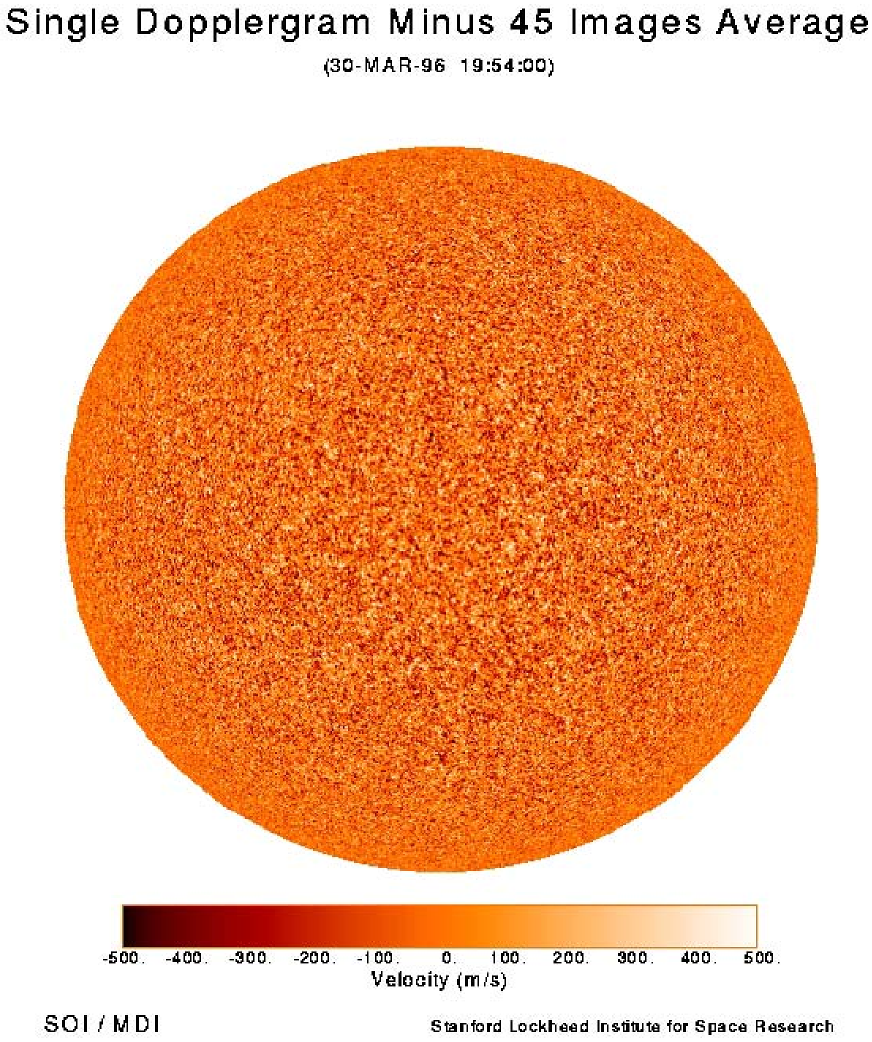
radiative interior

core

99% $\boldsymbol{L}$

49% $\boldsymbol{M}$

Solar oscillations (1961-) and seismic sounding



Seismic observable:

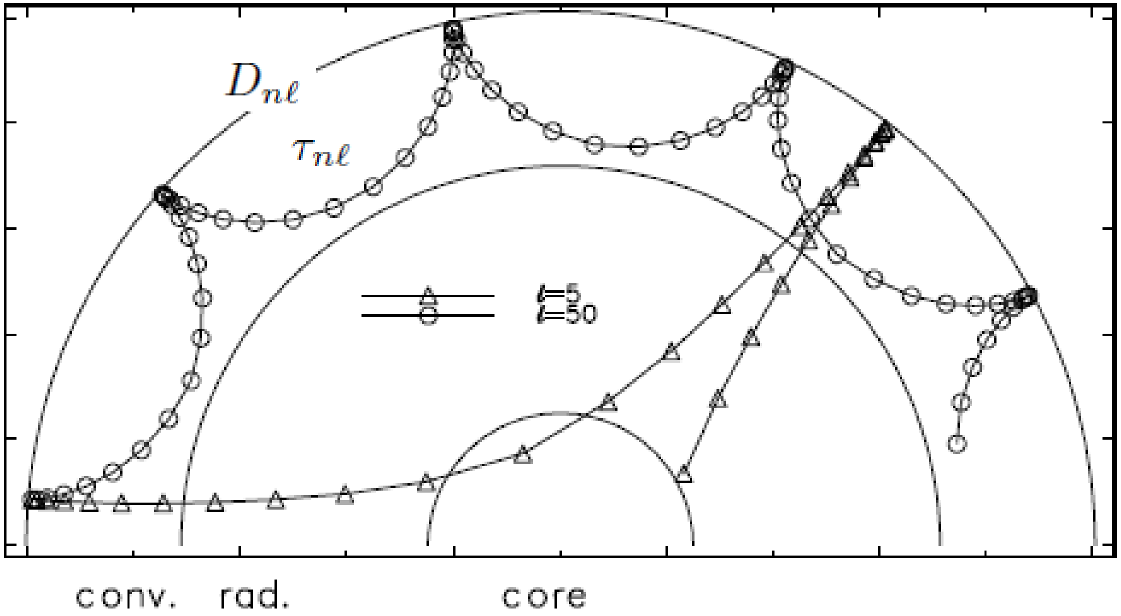
frequencies (1984 -)

propagation times (1997 -)

Acousitc waves $\omega^2 = c^2\left[k_r^2 + \frac{\ell(\ell+1)}{r^2}\right]$

Group velocty $v_g = \frac{\partial\omega}{\partial k}$

Local heioseismology: $\tau_{n\ell}$ $D_{n\ell}$



Standing waves = modes:

Global heioseismology: $\omega_{n\ell m}$

Modes of stellar oscillations

rotation neglected

$$v_{r,n\ell m} = \omega_{n\ell} R A_{n\ell m} \Re[y_{n\ell}(r) Y_\ell^m(\theta,\phi) \mathrm{e}^{\mathrm{i}(\psi_{n\ell m} - \omega_{n\ell} t)}] \mathrm{e}^{-\gamma_{n\ell} t} \qquad \omega = 2\pi\nu$$

n – radial order ***ℓ*** – angular degree ***m***– azimuthal order

$$y_{n\ell}(R) = 1$$

slow rotation

$$\omega_{n\ell m} = \omega_{n\ell,0} + \int_0^R \int_0^1 \mathcal{R}_{n\ell m}(r,\mu)\Omega(r,\mu) d\mu dr \qquad \mu = \cos\theta$$

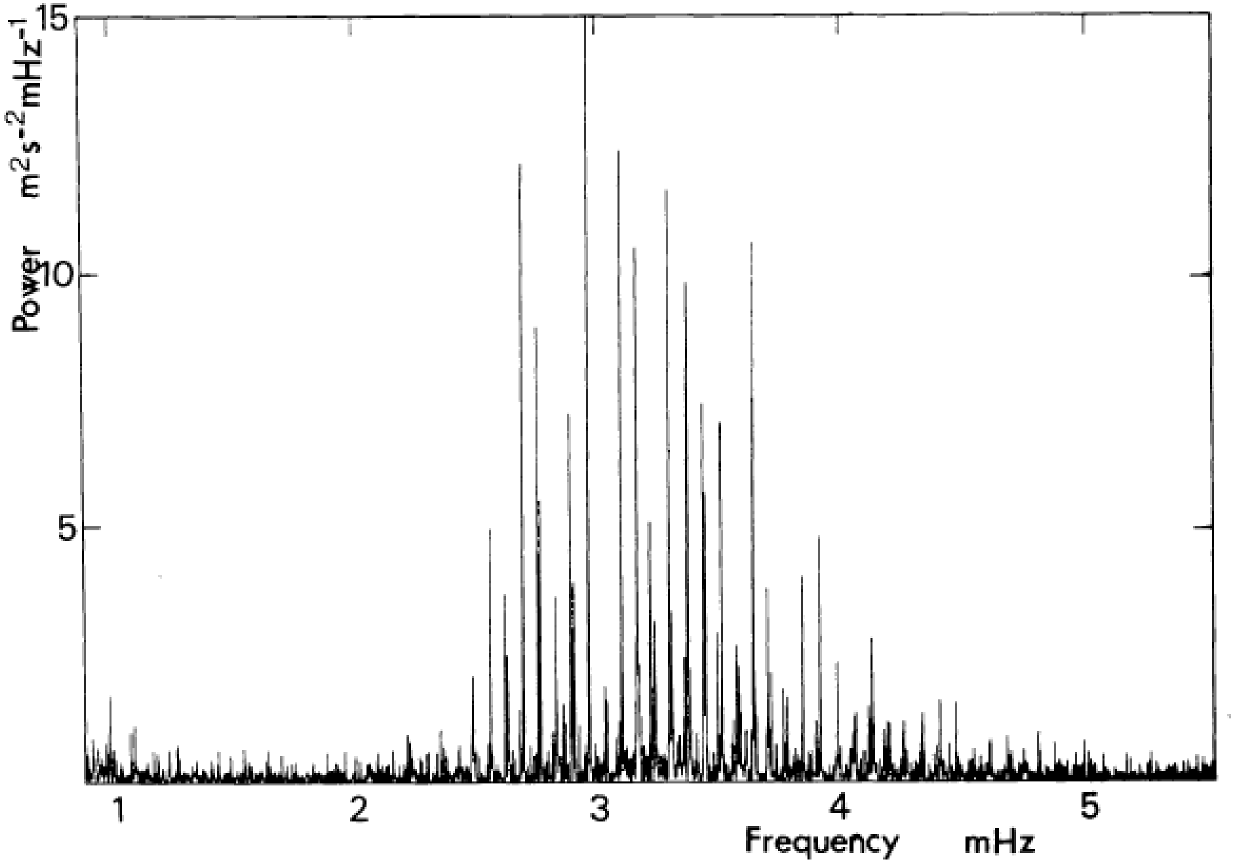
slow and uniform rotation

$$\omega_{n\ell m} = \omega_{n\ell,0} + m(1 - C_{n\ell})\Omega$$

p – and f – modes detected in the Sun

Low order high degree modes from the whole disc data

ℓ>> n, **ℓ=1** singlets and **ℓ=0&2** doublets



(Grec et al.1980)

Frequencies of higher degree p- and f- modes

$$\nu_{n\ell}(m) = \bar{\nu}_{n\ell} + \sum_{k=1}^{2\ell} a_{k,n\ell}\mathcal{P}_k^\ell(m)$$

$\mathcal{P}_k^\ell(m)$

orthogonal polynomials of $k \le \ell$ degree

$\ell \gg k$ $\mathcal{P}_k^\ell(m) \approx \ell P_\ell^m(m/\ell)$

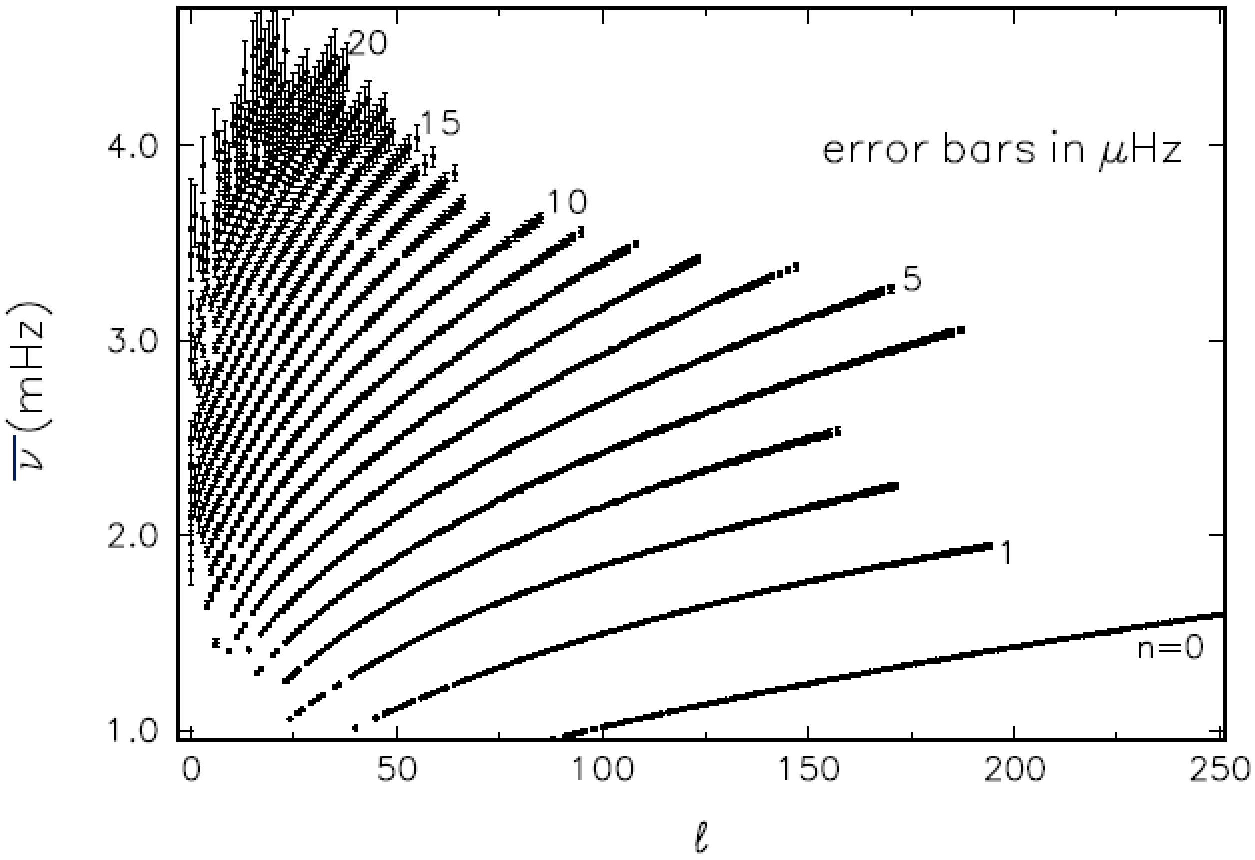
$2\pi\bar{\nu}_{n\ell} \approx \omega_{n\ell}$ the probe of internal structure

odd ***a*** –coefficients: the probe of differential rotation

even ***a*** – coefficients the probe of asphericity

The centroid frequencies from SOHO MDI data

p_n - and f (p_0) - modes



Scherrer et al. 1996

The helioseismic inverse problem
1990 -

assume hydrostatic equilibrium and $\Gamma_1(\rho, p, Y)$

$$\Gamma_1 \equiv \left(\frac{\partial \ln T}{\partial \ln p}\right)_{s,\boldsymbol{X}} \approx 5/3$$

$\rho_{\mathrm{model}}(r)$

$p_{\mathrm{model}}(r)$ → $\nu_{n\ell,\mathrm{model}}$ $u = p/\rho$

$\Gamma_{1,\mathrm{model}}(r)$

$$\left(\frac{\delta\nu}{\nu}\right)_{n\ell} = \int_0^R \mathcal{K}^u_{n\ell}\frac{\delta u}{u}dr + \mathcal{J}_{n\ell}\delta Y_{\mathrm{env}} + \frac{F_{\mathrm{surf}}(\nu)}{I_{n\ell}} - \frac{3}{2}\frac{\delta R}{R}$$

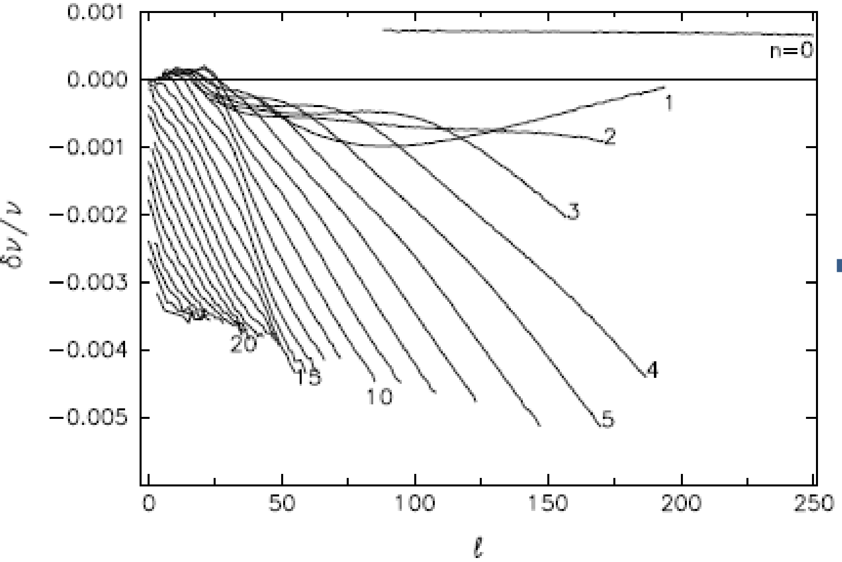
$\delta\nu = \nu_\odot - \nu_{\mathrm{model}}$ →

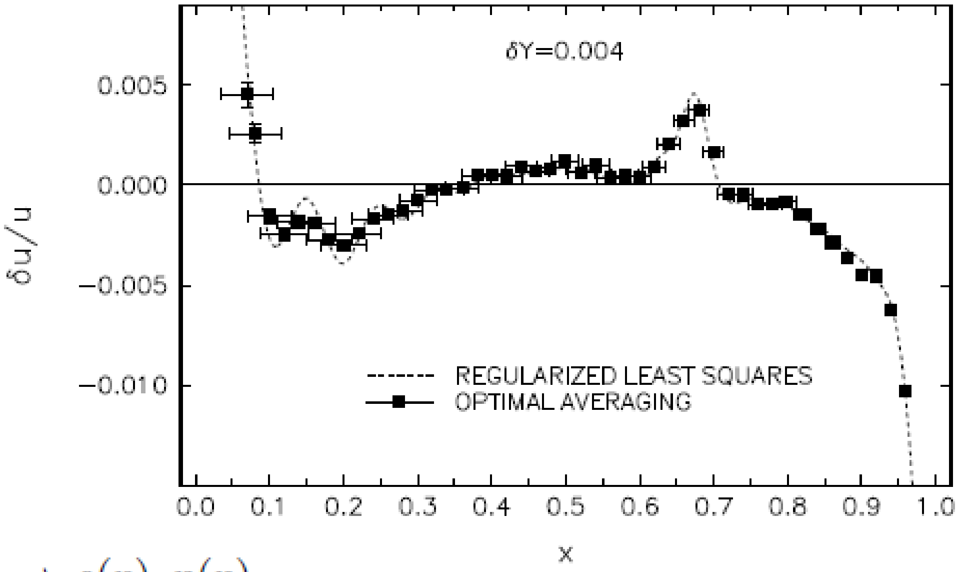
$\delta u = u_\odot - u_{\mathrm{model}}$

$\delta Y = Y_\odot - Y_{\mathrm{model}}$

$\delta R = R_\odot - R_{\mathrm{model}}$

Inversion by two methods





$$\delta u(r) \to \rho(r), p(r)$$

very small corrections in 1990s, larger after 2003 !

δY_{env}

most accurate measurment of the He abundance

$$\frac{\delta R}{R} = -4.3 \times 10^{-4}$$ solar diameter corrected

Probing internal rotation
1984 -

photosphere

$$\frac{\Omega(R,\mu)}{2\pi} = 450 - 77\mu^2 - 54\mu^4 \text{ nHz}$$

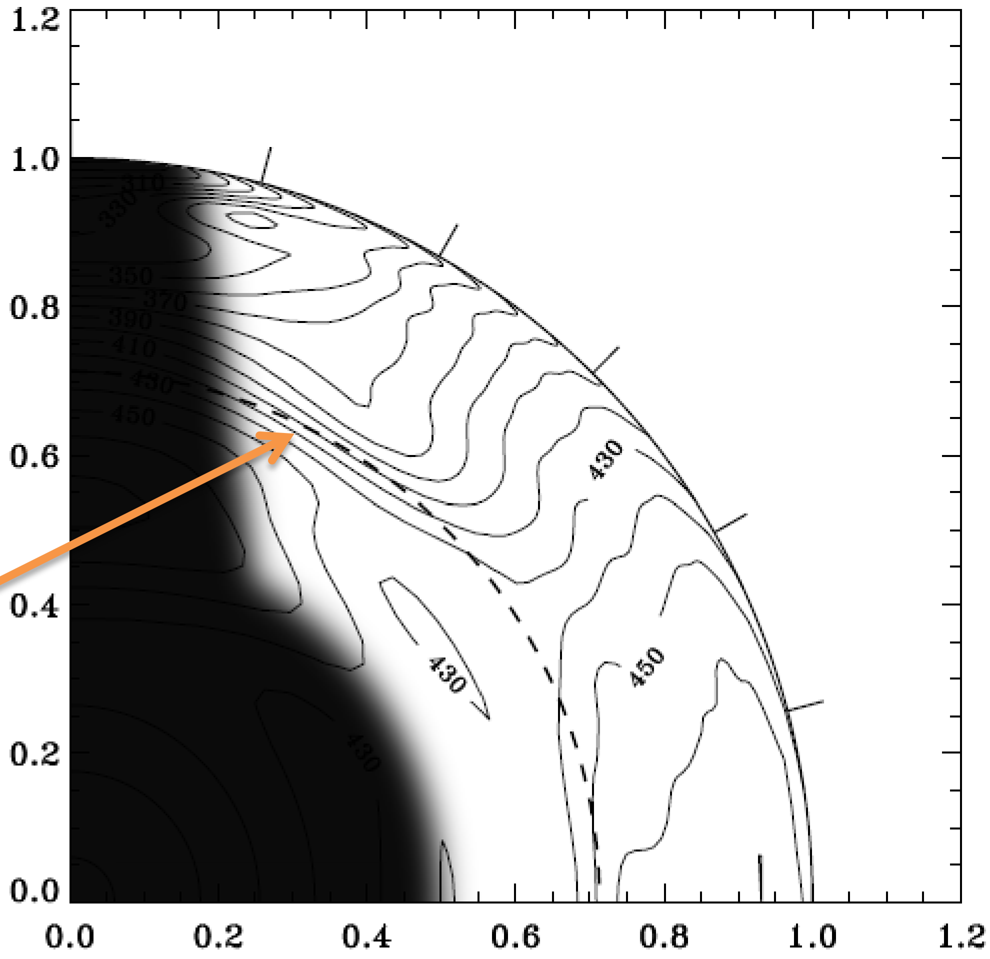
$$\Omega(r,\mu) = \sum_{k=0} \widetilde{\Omega}_{2k+1}(r)\frac{dP_{2k+1}(\mu)}{d\mu}$$

$$\omega_{n\ell m} = \omega_{n\ell,0} + \int_0^R \int_0^1 \mathcal{R}_{n\ell m}(r,\mu)\Omega(r,\mu)d\mu dr$$

$$\int_0^1 \mathcal{R}_{n\ell m}(r,\mu)\frac{dP_{2k+1}(\mu)}{d\mu} = \mathcal{P}_k^{\ell}(m)\mathcal{K}_{n\ell}^{\mathrm{rot}}(r)$$

$$\int_0^R \mathcal{K}_{\mathrm{rot},n\ell}(x)\widetilde{\Omega}_{2k+1}(r)dr = 2\pi a_{2k+1,n\ell}$$

Internal rotation from SOHO MDI data



the tachocline –
site of toroidal field
generation

Shou et al. 1999

New photospheric metal abundance
3D nonLTE atmosphere model

$$(Z/X)_{\mathrm{phot}}= \begin{cases} 0.0229 \text{ (Grevesse \& Sauval 1998)} \\ 0.0181 \text{ (Asplund et al. 2005)} \\ 0.0165 \text{ (Asplund et al. 2009)} \end{cases}$$

spoils good agreement of the standard solar model
wIth seismic sounding

from Bi et al. (2011)

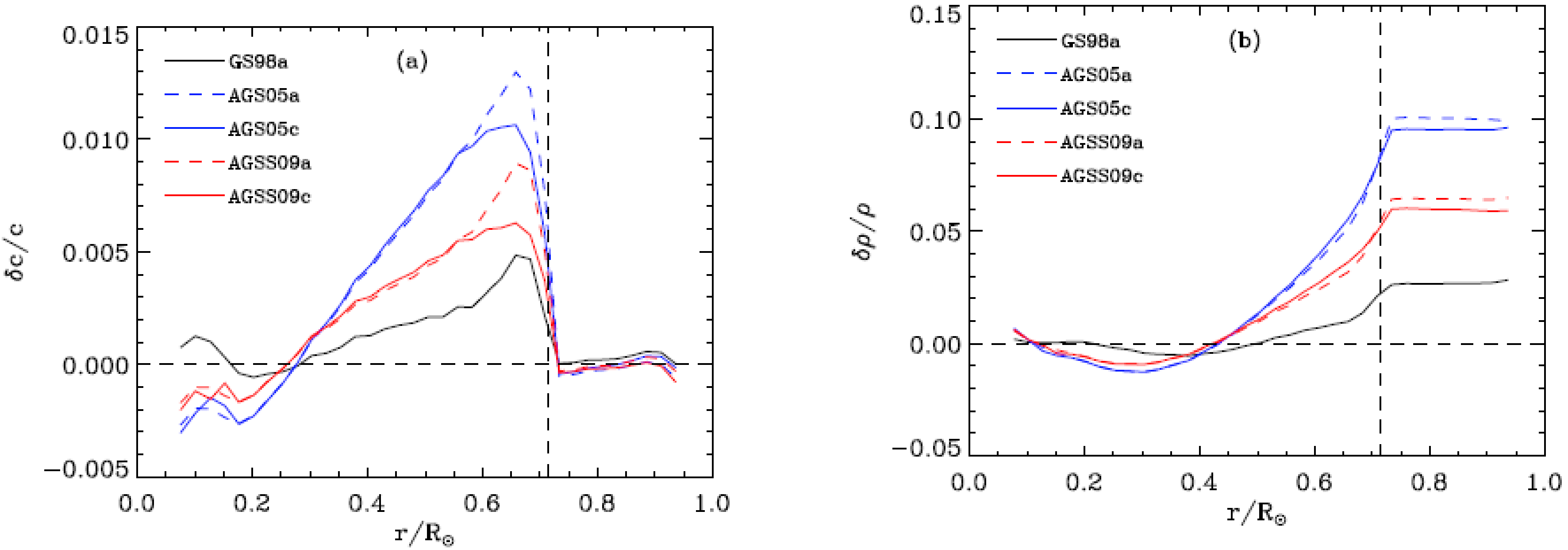


Table 1. Characteristics of the Calibrated Solar Models

Model	$(Z/X)_s$	Z_s	Y_s	$R_{cz}/R_\odot$	$<\delta c/c>$	$<\delta\rho/\rho>$	Y_c	Z_c	Y_{ini}	Z_{ini}	α_{MLT}
GS98a	0.0229	0.0169	0.246	0.715	0.0012	0.008	0.644	0.0198	0.277	0.0188	2.12
AGS05a	0.0165	0.0125	0.230	0.728	0.0030	0.034	0.623	0.0148	0.261	0.0140	2.08
AGS05b	0.0165	0.0124	0.239	0.727	0.0028	0.035	0.622	0.0146	0.269	0.0139	2.04
AGS05c	0.0165	0.0124	0.237	0.726	0.0028	0.033	0.621	0.0146	0.260	0.0139	2.05
AGSS09a	0.0181	0.0136	0.236	0.723	0.0020	0.022	0.631	0.0160	0.267	0.0152	2.12
AGSS09b	0.0181	0.0134	0.245	0.722	0.0019	0.023	0.630	0.0158	0.268	0.0150	2.07
AGSS09c	0.0181	0.0135	0.243	0.721	0.0017	0.021	0.630	0.0158	0.266	0.0150	2.09

[a] Solar models with diffusion.

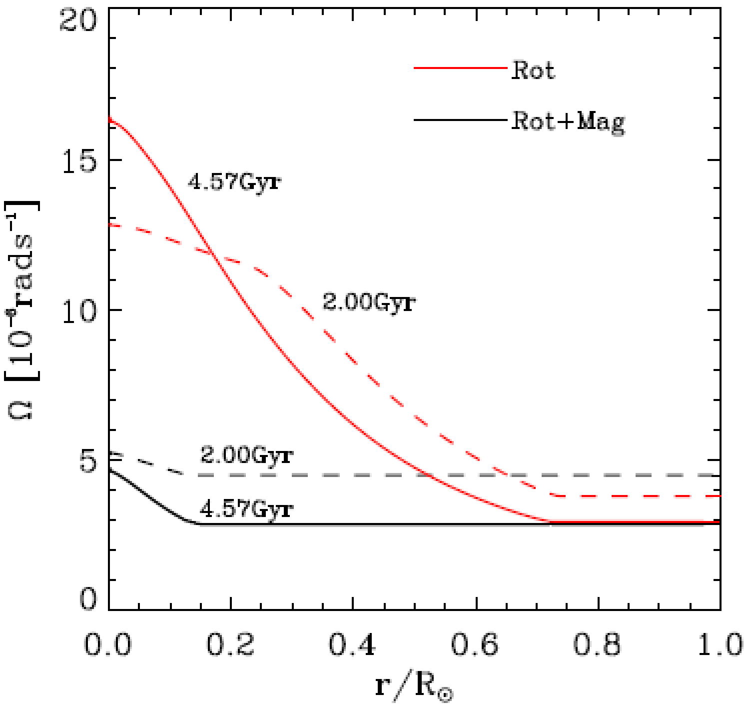
[b] Solar models with diffusion and rotation.

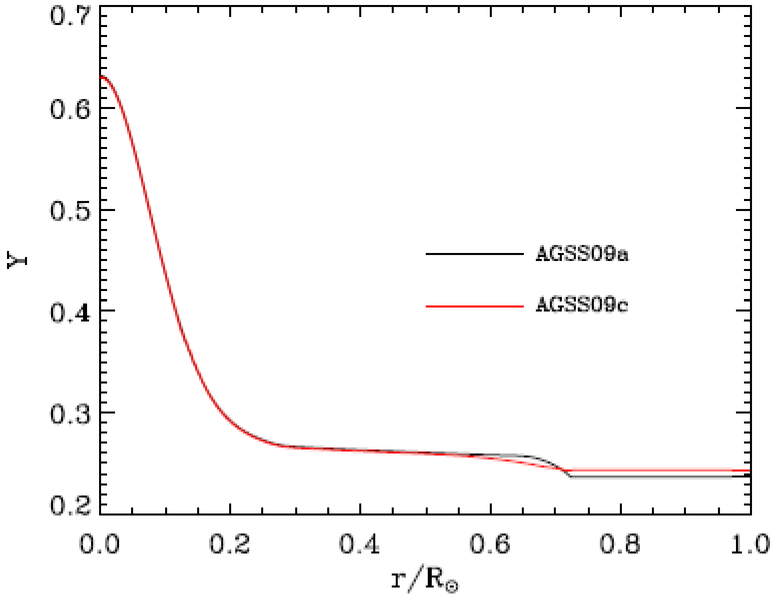
[c] Solar models with diffusion, rotation and magnetic fields.

Macroscopic mixing lowers the discrepancy a little

$$\frac{\partial \Omega}{\partial t} = \frac{1}{\rho r^2}\frac{\partial}{\partial r}\left(\rho r^2 \lambda_\Omega \frac{\partial \Omega}{\partial r}\right)$$

$$\frac{\partial \boldsymbol{X}}{\partial t} = \frac{1}{\rho r^2}\frac{\partial}{\partial r}\left(\rho r^2 \lambda_X \frac{\partial \boldsymbol{X}}{\partial r}\right)$$





Magnetic field needed for consitency with the seismic **Ω (*r*)**

Problems and prospect

All measurements of the neutrinto flux consistent with standard models
Prospects for contraints on ***T*** and ***Z*** i n the core

Small but significant corrections needed for conisitency with helioseismic inversions
Possible solutions: (1) revision of photospheric element abundance
(2) revision in opacity calculation

Macroscopic mechanism(s) of element and angular momentum transport
Tools: 3D simulations, Local helioseismology

Solar activity cycle , the site of dynamo, the cause of long-time changes.